Mathletics
Illinois Program of Studies
Skill Quests

Grades 7 – 8
January, 2023

Mathletics

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Grade 7

1 Ratios and Proportional Relationships

1.1 Analyze proportional relationships and use them to solve real-world and mathematical problems

Outcome	Quests	Content
Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	Unit rates with fractions	Solving unit rate problems involving fractions
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	Identify proportional relationships	Identifying proportional relationships
Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	Constant of proportionality	Identifying the constant of proportionality
Represent proportional relationships by equations.	Represent proportional relationships	Representing proportional relationships: equations
Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	Graphs of proportional relationships	Interpreting graphs of proportional relationships
Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	Ratio & percent problems	Solving multi-step ratio & percent problems

2 Number System

2.1 Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers

Outcome	Quests	Content
Describe situations in which opposite quantities combine to make 0.	Opposites	Describing situations involving opposites
Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	Add rational numbers	Opposites & absolute value
		Adding rational numbers
		Adding positive & negative fractions
		Adding positive & negative decimals
		Adding integers
Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	Subtract rational numbers	Subtracting rational numbers: adding the inverse
		Subtracting positive & negative fractions
		Subtracting positive & negative decimals
		Subtracting integers
		Subtracting rational numbers: absolute value
Apply properties of operations as strategies to add and subtract rational numbers.	Rational numbers: addition properties	Adding & subtracting rational numbers: properties
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing realworld contexts.	Multiply rational numbers	Multiplying rational numbers
		Multiplying positive & negative fractions
		Multiplying positive & negative decimals
		Multiplying integers
		Products of rational numbers: real-world contexts
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are	Divide integers	Dividing integers
		Quotients of rational numbers: real-world contexts

integers then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts.		
Apply properties of operations as strategies to multiply and divide rational numbers.	Rational numbers: properties	Multiply & divide rational numbers: properties
Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	Convert rational numbers to decimals	Use long division to convert rationals to decimals
Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	Rational numbers problems: 4 operations	Rational numbers problems: 4 operations

3 Expressions and Equations

3.1 Use properties of operations to generate equivalent expressions

Outcome	Quests	Content
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	Linear expressions: properties	Simplifying algebraic expressions: add & subtract
		Distributive property: algebraic expressions
		Factoring algebraic expressions
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	Interpret expressions	Rearranging expressions to interpret quantities

3.2 Solve real-life and mathematical problems using numerical and algebraic expressions and equations

Outcome	Quests	Content
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	Problems with rational numbers	Solving problems with rational numbers
		Converting terminating decimals
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	Solve 2-step equations	Solving 2-step equations: word problems
		2-step equations, positive integer coefficients
		2-step equations, integer coefficients
		2-step equations, positive rational coefficients
		2-step equations, rational coefficients

		2-step equations, distributive property
Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	Solve 2-step inequalities	Creating & solving 2-step inequalities
		Representing inequalities
		Graphing the solution of an inequality
		Solving 2-step inequalities

4 Geometry

4.1 Draw, construct, and describe geometrical figures and describe the relationships between them

Outcome	Quests	Content
Draw, construct, and describe geometrical figures and describe the relationships between them. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Scale drawings	Scale drawings
Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Construct triangles	Triangle inequality theorem Constructing triangles with given conditions
Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	Cross sections of 3-D figures	Describing cross sections of 3-D figures

4.2 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

Outcome	Quests	Content
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Circles: area & circumference	Finding the area of a circle Introducing the parts of a circle Finding the circumference of a circle
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step	Use angle facts to solve problems	Supplementary angles Complementary angles Adjacent angles

problem to write and solve simple equations for an unknown angle in a figure.		Vertical angles
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Area, volume & surface area	Area: polygons
		Solving real-life problems: area of polygons
		Volume: right prisms
		Surface area: rectangular & triangular prisms

5 Statistics and Probability

5.1 Use random sampling to draw inferences about a population

Outcome	Quests	Content
Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	Understand sampling	Understanding sampling
Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	Draw inferences from samples	Drawing inferences from samples

5.2 Draw informal comparative inferences about two populations

Outcome	Quests	Content
Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	Compare data distributions	Comparing data distributions
Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	Draw comparative inferences	Drawing comparative inferences

5.3 Investigate chance processes and develop, use, and evaluate probability models

Outcome	Quests	Content
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	Introduction to probability	Introducing probability
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	Probability of chance events	Probability of chance events: relative frequency
Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.	Determine the probability of events	Theoretical probability
		Predicting outcomes of chance experiments
		Finding the complement of an event
Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	Observe frequencies in data	Finding the approximate probability
		Comparing observed frequency & expected frequency
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	Probability: compound events	Investigating mutually exclusive events
		Calculating probabilities of compound events
Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	Sample spaces for compound events	Representing sample spaces & identifying outcomes

Design and use a simulation to generate frequencies for compound events.	Independent & dependent compound events	Independent & dependent compound events

| Design and use a simulation to generate frequencies for compound | Independent & dependent compound | Independent & dependent compound events |

Grade 8

1 Number System

1.1 Know that there are numbers that are not rational, and approximate them by rational number

Outcome	Quests	Content
Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0's or eventually repeat. Know that other numbers are call irrational.	Rational & irrational numbers	Describing properties of irrational numbers
		Classifying real numbers
		Converting repeating decimals to rational numbers
		Repeating & terminating decimals as fractions
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2).	Approximate irrational numbers	Comparing irrational numbers
		Locating irrational numbers on a number line
		Approximating the value of an irrational number
		Finding square roots of non-perfect squares

2 Expressions and Equations

2.1 Work with radicals and integer exponents

Outcome	Quests	Content
Know and apply the properties of integer exponents to generate equivalent numerical expressions.	Properties of integer exponents	Using exponent notation
		Product of powers, numerical base
		Product of powers, algebraic base
		Quotient of powers, numerical base
		Quotient of powers, algebraic base
		Power of a power, numerical base
		Power of a power, algebraic base
		Zero exponents, numerical base
		Zero exponents, algebraic base
		Zero exponents, algebraic base
		Negative exponents, algebraic base
		Simplifying expressions, numerical base
		Simplifying expressions, algebraic base
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	Square & cube roots	Investigating square roots & cube roots
		Squares & square roots
		Evaluating expressions with square & cube roots
		Square roots of fractions & decimals
		Cubes & cube roots
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	Write numbers in scientific notation	Introducing scientific notation
		Converting scientific notation to standard form
		Converting standard form to scientific notation
Perform operations with numbers expressed in scientific notation, including problems where both	Calculations in scientific notation	Calculations in scientific notation

decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	

2.2 Understand the connections between proportional relationships, lines, and linear equations

Outcome	Quests	Content
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	Proportional relationships	Graphing proportional relationships
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	Understand slope & y-intercept	Using similar triangles to understand slope
		Writing equations of proportional relationships
		Writing equations of nonproportional relationships
		Identifying the slope in an equation or graph
		Identifying the y-intercept on a graph
		Graphing equations in slope-intercept form
		Graphing equations not in slope-intercept form
		Finding the y-intercept algebraically

2.3 Analyze and solve linear equations and pairs of simultaneous linear equations

Outcome	Quests	Content
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by	Solution types of linear equations	Solution types of linear equations

successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).		
Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	Solve linear equations	Solving 3-step linear equations
		Solving linear equations, variables on both sides
		Solving linear equations, distributive property
		Using substitution to check solutions
Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	Identify solutions, systems of equations	Identifying solutions, systems of equations
Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.	Solve systems of equations	Solving systems of equations graphically
		Solving systems of equations using elimination
		Solving systems of equations using substitution
		Checking the solution of a system of equations
Solve real-world and mathematical problems leading to two linear equations in two variables.	Write & solve systems of equations	Writing & solving systems of equations

3 Functions

3.1 Define, evaluate, and compare functions

Outcome	Quests	Content
Define, evaluate, and compare functions. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	Identify functions	Identifying functions
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	Compare functions	Comparing functions represented in different ways
Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	Interpret $y = mx + b$ as linear	Represent linear relationships in different forms

3.2 Use functions to model relationships between quantities

Outcome	Quests	Content
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	Rate of change & initial value	Rate of change & initial value
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the	Distance-time graphs	Distance-time graphs

qualitative features of a function that has been described verbally.		

4 Geometry

4.1 Understand congruence and similarity using physical models, transparencies, or geometry software

Outcome	Quests	Content
Verify experimentally the properties of rotations, reflections, and translations: -- a. Lines are taken to lines, and line segments to line segments of the same length. -- b. Angles are taken to angles of the same measure. -- c. Parallel lines are taken to parallel lines.	Introduction to rigid transformations	Translating points on the coordinate plane
		Reflecting points across the x- or y-axis
		Rotating points about the origin
Lines are taken to lines, and line segments to line segments of the same length.	Preserved properties: length	Preserved properties: length
Angles are taken to angles of the same measure.	Preserved properties: angles	Preserved properties: angles
Parallel lines are taken to parallel lines.	Preserved properties: parallel lines	Preserved properties: parallel lines
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Congruency: rigid transformations	Congruency: rigid transformations
Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.	Transformations, coordinates	Dilations, coordinates
		Translations, coordinates
		Rotations, coordinates
		Reflections, coordinates
		Sequences of transformations
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	Similarity: transformations	Introducing similarity
		Similarity: transformations
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when	Triangles & angle relationships	Angle sum theorem
		Exterior angle theorem
		Angle relationships: parallel lines, transversal

Outcome		Content
parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.		Using scale to analyze similar triangles
		Identifying similar triangles

4.2 Understand and apply the Pythagorean Theorem

Outcome	Quests	Content
Explain a proof of the Pythagorean Theorem and its converse.	The Pythagorean Theorem & its converse	Identifying the hypotenuse, right triangles
		Identifying right triangles, Pythagorean Theorem
		Pythagorean triples
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Apply the Pythagorean Theorem	Pythagorean Theorem: missing short side
		Pythagorean Theorem: missing hypotenuse
		Pythagorean Theorem: missing side
		Pythagorean Theorem in 2-D & 3-D
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	Distance between two points	Finding the distance between two points

4.3 Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

Outcome	Quests	Content
Know the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	Volume: cones, cylinders & spheres	Volume: cones
		Volume: cylinders
		Volume: spheres

5 Statistics and Probability

5.1 Investigate patterns of association in bivariate data

Outcome	Quests	Content
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Use & interpret scatter plots	Using & interpreting scatter plots
Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	Estimate the line of best fit	Estimating the line of best fit
Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	Interpret the line of best fit	Interpreting the line of best fit
Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	Two-way tables	Constructing & interpreting two-way tables

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