Algebra

Expressions, Quadratics, Logarithms, Radicals and Complex Numbers

Expanding

$(x+y)^2 = x^2+2xy+y^2 \qquad (x-y)^2 = x^2-2xy+y^2$

Rearranging these we get the following results:

$x^2+y^2 = (x+y)^2-2xy \qquad x^2+y^2=(x-y)^2+2xy$

$(x+y)^3 = x^3+3xy(x+y)+y^3 = x^3+3x^2y+3xy^2+y^3$

Rearranging this will get:

$x^3+y^3=(x+y)^3-3xy(x+y)=(x+y)^3-3x^2y-3xy^2$

$(x-y)^3 = x^3-3xy(x-y)-y^3 = x^3-3x^2y+3xy^2-y^3$

Rearranging this will get:

$x^3-y^3=(x-y)^3+3xy(x-y)=(x-y)^3+3x^2y-3xy^2$

Logarithms

If $y=\log_b x$ then $x=b^y$

$\log_b b = 1$ (since $b=b^1$)

$\log_b x^a = a\log_b x$

$\log_b 1 = 0$ (since $1=b^0$)

$\log_b b^x = x$ (since $x\log_b b = x\times 1$)

$b^{\log_b x}=x$

$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$

if b, x, y are positive real numbers and $b\neq 1$

$\log_a x = \frac{\log_b x}{\log_b a}$

if a, b, x are positive real numbers and $a\neq 1$ and $b \neq 1$

$\log_b a = \frac{1}{\log_a b}$

where a and b are positive real numbers and $a\neq 1, b\neq 1$

If b, x, y are positive real numbers and $b\neq 1$ and $\log_b x = \log_b y$, then $x=y$

$\log_b xy = \log_b x + \log_b y$

if b, x, y are positive real numbers and $b \neq 1$

Quadratics

If $ax^2+bx+c=0$ and $a\neq 0$, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

b^2-4ac is called the discriminant $(\triangle)$.

If $\triangle = b^2-4ac=0$, The roots are real and equal.

If $\triangle = b^2-4ac>0$, The roots are real and unique.

If $\triangle = b^2-4ac<0$, The roots are non-real.

If α and β are the roots of $ax^2+bx+c=0$, $a\neq 0$

$$\alpha+\beta=\frac{-b}{a} \qquad \alpha\beta=\frac{c}{a}$$

The quadratic equation with roots α and β is:

$$(x-\alpha)(x-\beta)=0 \qquad \therefore x^2-\underset{\substack{\uparrow\\ \text{Sum of}\\ \text{the roots}}}{(\alpha+\beta)}x+\underset{\substack{\uparrow\\ \text{Product of}\\ \text{the roots}}}{\alpha\beta}=0$$

Factoring

$a^2-b^2=(a+b)(a-b)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\ \cdots\ +b^{n-1})$

$a^{2n}-b^{2n}=(a^n-b^n)(a^n+b^n)$

Radical (Surd) Laws

$\sqrt{a}=a^{\frac{1}{2}}$ for $a\geq 0$

$\sqrt[n]{a}=a^{\frac{1}{n}}$; $a\geq 0$ for positive values of n.

$\sqrt[n]{a^m}=a^{\frac{m}{n}}$; $a\geq 0$ for positive values of n.

$\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=a^{\frac{1}{mn}}$; $a\geq 0$ for positive values of nm.

$\sqrt[n]{ab}=\sqrt[n]{a}\times\sqrt[n]{b}=(ab)^{\frac{1}{n}}$; $a,b\geq 0$ for positive values of n.

$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\left(\frac{a}{b}\right)^{\frac{1}{n}}$; $a,b\geq 0$ for positive values of n.

$\sqrt[n]{a^n}=a$; for odd values of n.

$\sqrt[n]{a^n}=|a|$; for even values of n.

Complex Numbers

$i=\sqrt{-1} \qquad i^2=-1 \qquad \sqrt{-a}=i\sqrt{a}\ ;\ a\geq 0$

If $a+bi=0$ where $i=\sqrt{-1}$, then $a=b=0$

If $a+bi=x+yi$ where $i=\sqrt{-1}$, then $a=x$ and $b=y$

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

$$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$$

$$(a+bi)(a-bi)=a^2+b^2$$

$$\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\times\frac{c-di}{c-di}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$|a+bi|=\sqrt{a^2+b^2}$

$\overline{a+bi}=a-bi$

$\overline{a-bi}=a+bi$

$\overline{a+bi}\,(a+bi)=|a+bi|^2$

$$\frac{1}{a+bi}=\frac{a-bi}{(a+bi)(a-bi)}=\frac{a-bi}{a^2+b^2}$$

De Moivre's Theorem:

$$[r(\cos\theta+i\sin\theta)]^n=r^n(\cos n\theta+i\sin n\theta)$$

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