

Algebra

Expressions, Quadratics, Logarithms, Radicals and Complex Numbers

Expanding

$$(x + y)^2 = x^2 + 2xy + y^2 \quad (x - y)^2 = x^2 - 2xy + y^2$$

Rearranging these we get the following results:

$$x^2 + y^2 = (x + y)^2 - 2xy \quad x^2 + y^2 = (x - y)^2 + 2xy$$

$$(x + y)^3 = x^3 + 3xy(x + y) + y^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

Rearranging this will get:

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y) = (x + y)^3 - 3x^2y - 3xy^2$$

$$(x - y)^3 = x^3 - 3xy(x - y) - y^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Rearranging this will get:

$$x^3 - y^3 = (x - y)^3 + 3xy(x - y) = (x - y)^3 + 3x^2y - 3xy^2$$

Logarithms

If $y = \log_b x$ then $x = b^y$ $\log_b b = 1$ (since $b = b^1$)

$\log_b x^a = a \log_b x$ $\log_b 1 = 0$ (since $1 = b^0$)

$\log_b b^x = x$ (since $x \log_b b = x \times 1$) $b^{\log_b x} = x$

$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

if b, x, y are positive real numbers and $b \neq 1$

$\log_a x = \frac{\log_b x}{\log_b a}$

if a, b, x are positive real numbers and $a \neq 1$ and $b \neq 1$

$\log_b a = \frac{1}{\log_a b}$

where a and b are positive real numbers and $a \neq 1, b \neq 1$

If b, x, y are positive real numbers and $b \neq 1$ and

$\log_b x = \log_b y$, then $x = y$

$\log_b xy = \log_b x + \log_b y$

if b, x, y are positive real numbers and $b \neq 1$

Quadratics

If $ax^2 + bx + c = 0$ and $a \neq 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$ is called the discriminant (Δ).

If $\Delta = b^2 - 4ac = 0$, The roots are real and equal.

If $\Delta = b^2 - 4ac > 0$, The roots are real and unique.

If $\Delta = b^2 - 4ac < 0$, The roots are non-real.

If α and β are the roots of $ax^2 + bx + c = 0$, $a \neq 0$

$$\alpha + \beta = \frac{-b}{a} \quad \alpha\beta = \frac{c}{a}$$

The quadratic equation with roots α and β is:

$$(x - \alpha)(x - \beta) = 0 \quad \therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

\uparrow Sum of the roots
 \uparrow Product of the roots

Factoring

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$$

$$a^{2n} - b^{2n} = (a^n - b^n)(a^n + b^n)$$

Radical (Surd) Laws

$$\sqrt[n]{a} = a^{\frac{1}{n}} \text{ for } a \geq 0$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}; a \geq 0 \text{ for positive values of } n.$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}; a \geq 0 \text{ for positive values of } n.$$

$$\sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{a} = a^{\frac{1}{n}}; a \geq 0 \text{ for positive values of } nm.$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} = (ab)^{\frac{1}{n}}; a, b \geq 0 \text{ for positive values of } n.$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}; a, b \geq 0 \text{ for positive values of } n.$$

$$\sqrt[n]{a^n} = a; \text{ for odd values of } n.$$

$$\sqrt[n]{a^n} = |a|; \text{ for even values of } n.$$

Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}; a \geq 0$$

If $a + bi = 0$ where $i = \sqrt{-1}$, then $a = b = 0$

If $a + bi = x + yi$ where $i = \sqrt{-1}$, then $a = x$ and $b = y$

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

$$|a + bi| = \sqrt{a^2 + b^2} \quad \overline{a + bi} = a - bi$$

$$\overline{a - bi} = a + bi$$

$$\overline{a + bi}(a + bi) = |a + bi|^2$$

$$\frac{1}{a + bi} = \frac{a - bi}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$$

De Moivre's Theorem:

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$



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Algebra

Operations, Inequalities and Exponent Laws

Arithmetic Rules in Algebra

Commutative: $a + b = b + a$
 $a \times b = b \times a$

Associative: $(a + b) + c = a + (b + c)$
 $a \times (b \times c) = (a \times b) \times c$

Distributive: $a(b + c) = ab + ac$
 $a(b - c) = ab - ac$

Arithmetic Operations in Algebra

$$ab + ac = a(b + c)$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{ad + ac}{a} = d + c, a \neq 0$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}, c \neq 0$$

$$ab - ac = a(b - c)$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{ad - ac}{a} = d - c, a \neq 0$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}, c \neq 0$$

$$\frac{a}{b} \div c = \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$$

$$\frac{a-b}{c-d} = \frac{-(b-a)}{-(d-c)} = \frac{b-a}{d-c}$$

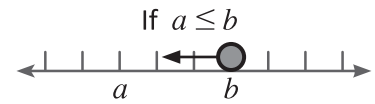
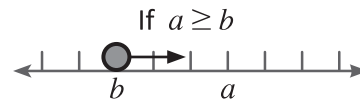
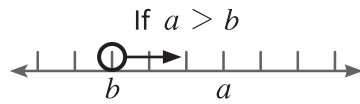
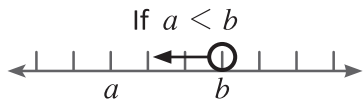
$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

$$\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

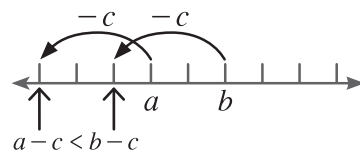
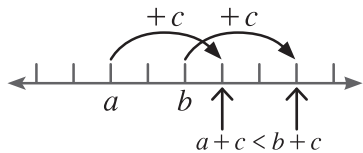
$$a \div \frac{b}{c} = \frac{a}{\left(\frac{b}{c}\right)} = a \times \frac{c}{b} = \frac{ac}{b}$$

$$a \times \frac{b}{c} = \frac{ab}{c}$$

Inequalities



If $a < b$, then $a + c < b + c$ and $a - c < b - c$

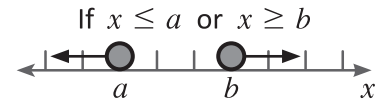
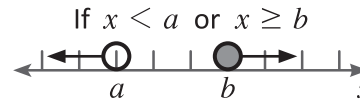
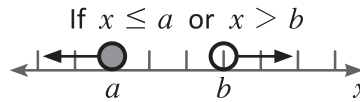
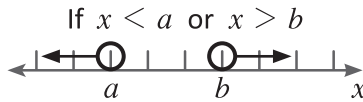
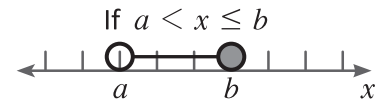
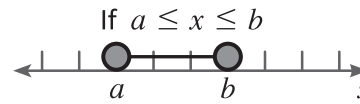
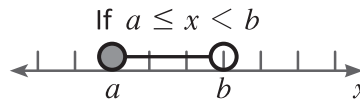
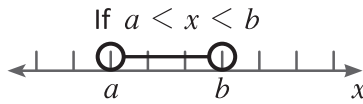


If $a < b$, and $c > 0$ then:

- $ac < bc$
- $\frac{a}{c} < \frac{b}{c}$

If $a < b$, and $c < 0$ then:

- $ac > bc$
- $\frac{a}{c} > \frac{b}{c}$



Absolute Value

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

$$|a| = |-a|$$

$$|a| \geq 0$$

$$|a + b| \leq |a| + |b|$$

$$|ab| = |a| \times |b|$$

$$\left|\frac{a}{b}\right| \geq \frac{|a|}{|b|} = |a \div b| = |a| \div |b|$$

Exponent Laws and their Variations

$$a^n = a \times a \times a \dots n \text{ times} \quad a^1 = a \quad a^0 = 1, \text{ where } a \in R, a \neq 0$$

If $a^m = a^n$ and $a \neq \pm 1$, and $a \neq 0$, then $m = n$

If $a^m = b^m$ and $m \neq 0$, then $a = \pm b$

$$(a^m)^n = a^{m \times n} = a^{mn} \quad (ab)^m = a^m \times b^m = a^m b^m \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^{-m} = \frac{1}{a^m}, a \neq 0 \quad \frac{1}{a^{-m}} = 1 \div \frac{1}{a^m} = 1 \times \frac{a^m}{1} = a^m, a \neq 0$$

$$a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$a^{-\left(\frac{m}{n}\right)} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{(a^{\frac{1}{n}})^m} = \frac{1}{(a^m)^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a^m}} = \frac{1}{(\sqrt[n]{a})^m}$$

$$a^m \times a^n = a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n} \text{ if } m > n$$

$$= 1 \text{ if } m = n$$

$$= \frac{1}{a^{n-m}} \text{ if } m < n; a \in R, a \neq 0$$

$$\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{\left(\frac{1}{a^m}\right)}{\left(\frac{1}{b^m}\right)} = \frac{1}{a^m} \div \frac{1}{b^m} = \frac{1}{a^m} \times \frac{b^m}{1} = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$$

$$\left(\frac{a}{b}\right)^{\frac{m}{n}} = \frac{a^{\frac{m}{n}}}{b^{\frac{m}{n}}} = \frac{\sqrt[n]{a^m}}{\sqrt[n]{b^m}} = \frac{(\sqrt[n]{a})^m}{(\sqrt[n]{b})^m} = \left(\frac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^m = \left(\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}}\right)^m$$

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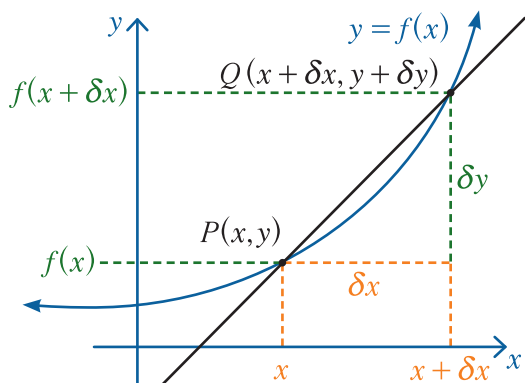
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Calculus

Differentiation

Basic Differentiation



Gradient of secant PQ is:

$$\frac{\delta y}{\delta x} = \frac{f(x + \delta x) - f(x)}{\delta x}$$

When $\delta x = 0$, PQ is a tangent to $y = f(x)$

$$\frac{d}{dx} f(x) = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

Basic Rules

$$\frac{d}{dx} f(x) = f'(x)$$

$$\frac{d}{dx} (ax) = a$$

$$\frac{d}{dx} (x) = 1$$

$$\frac{d}{dx} (c) = 0$$

$$\frac{d}{dx} [af(x)] = a \frac{d}{dx} [f(x)]$$

$$\frac{d}{dx} (ax^n) = nax^{n-1}$$

$$\frac{d}{dx} ([f(x)]^n) = n[f(x)]^{n-1} f'(x)$$

$$\frac{d}{dx} \left[\frac{1}{x^n} \right] = -nx^{-(n+1)} = -\frac{n}{x^{n+1}}$$

More Differentiation Rules

$$\frac{d}{dx} (f(x) \pm g(x)) = f'(x) \pm g'(x)$$

Chain Rule

$$\frac{d}{dx} [f(g(x))] = f'(g(x)) g'(x)$$

Product Rule

If $y = uv$, where $u = f(x)$ and $v = g(x)$

$$\frac{dy}{dx} = u'v + uv'$$

Quotient Rule

If $y = \frac{u}{v}$, where $u = f(x)$ and $v = g(x)$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

Reciprocal Rule

$$\frac{d}{dx} \left[\frac{1}{f(x)} \right] = \frac{-f'(x)}{[f(x)]^2}$$

Absolute value

$$\frac{d}{dx} |x| = \frac{x}{|x|}, \quad x \neq 0$$

Second Derivative

$$\text{If } f(x) = ax^n, \quad f'(x) = anx^{n-1}$$

$$\therefore \frac{d}{dx} f'(x) = f''(x) = an(n-1)x^{n-2}$$

Differentiation of Trigonometric Functions

$$\frac{d}{dx} (\sin x) = \cos x$$

$$\frac{d}{dx} (\sin[f(x)]) = f'(x) \cos[f(x)]$$

$$\frac{d}{dx} (\cos x) = -\sin x$$

$$\frac{d}{dx} (\cos[f(x)]) = -f'(x) \sin[f(x)]$$

$$\frac{d}{dx} (\tan x) = \sec^2 x$$

$$\frac{d}{dx} (\tan[f(x)]) = f'(x) \sec^2[f(x)]$$

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

$$\frac{d}{dx} (\sec[f(x)]) = f'(x) \sec[f(x)] \tan[f(x)]$$

$$\frac{d}{dx} (\cot x) = -\csc^2 x$$

$$\frac{d}{dx} (\cot[f(x)]) = -f'(x) \csc^2[f(x)]$$

$$\frac{d}{dx} (\csc x) = -\csc x \cot x$$

$$\frac{d}{dx} (\csc[f(x)]) = -f'(x) \csc[f(x)] \cot[f(x)]$$

Inverse Trigonometric Functions

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} (\sin^{-1} [f(x)]) = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$$

$$\frac{d}{dx} (\cos^{-1} [f(x)]) = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$$

$$\frac{d}{dx} (\tan^{-1} [f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

Hyperbolic Functions

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = 1 - \tanh^2 x$$

$$\frac{d}{dx} \operatorname{csch} x = -\coth x \operatorname{csch} x$$

$$\frac{d}{dx} \operatorname{sech} x = -\tanh x \operatorname{sech} x$$

$$\frac{d}{dx} \coth x = 1 - \coth^2 x$$

Exponential and Logarithmic Functions

$$\frac{d}{dx} [e^x] = e^x$$

$$\frac{d}{dx} (e^{f(x)}) = f'(x) e^{f(x)}$$

$$\frac{d}{dx} [a^x] = a^x \ln a$$

$$\frac{d}{dx} [a^{f(x)}] = a^{f(x)} \ln a f'(x)$$

$$\frac{d}{dx} [\log_e x] = \frac{d}{dx} [\ln(x)] = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx} [\log_b x] = \frac{1}{x \ln b}$$

$$\frac{d}{dx} [\ln|x|] = \frac{1}{x}$$

$$\frac{d}{dx} (\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx} (f(x)^{g(x)}) = f(x)^{g(x)} \left(\frac{g(x) f'(x)}{f(x)} + \ln(f(x)) g'(x) \right)$$

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Trigonometry

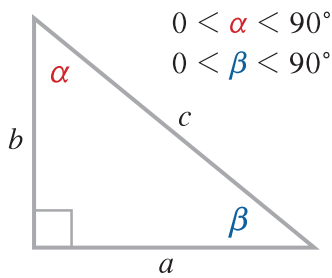
Identities

Degrees and Radians	Complementary Angles (Degrees)	Complementary Angles (Radians)
<p>If θ is an angle in degrees and α is the same angle in radians:</p> $\theta = \alpha \times \frac{180}{\pi} \quad \alpha = \theta \times \frac{\pi}{180}$	$\begin{aligned}\tan(90^\circ - \theta) &= \cot \theta \\ \cot(90^\circ - \theta) &= \tan \theta \\ \sin(90^\circ - \theta) &= \cos \theta \\ \csc(90^\circ - \theta) &= \sec \theta \\ \cos(90^\circ - \theta) &= \sin \theta \\ \sec(90^\circ - \theta) &= \csc \theta\end{aligned}$	$\begin{aligned}\tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta \\ \sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta \\ \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta\end{aligned}$
Odd or Even Identities	Pythagorean Identities	Periodic Identities
$\begin{aligned}\sin(-\theta) &= -\sin(\theta) \quad \csc(-\theta) = -\csc(\theta) \\ \cos(-\theta) &= \cos(\theta) \quad \sec(-\theta) = \sec(\theta) \\ \tan(-\theta) &= -\tan(\theta) \quad \cot(-\theta) = -\cot(\theta)\end{aligned}$	$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$	<p>If n is an integer</p> $\begin{aligned}\tan(\theta + \pi n) &= \tan \theta \quad \cot(\theta + \pi n) = \cot \theta \\ \sin(\theta + 2\pi n) &= \sin \theta \quad \csc(\theta + 2\pi n) = \csc \theta \\ \cos(\theta + 2\pi n) &= \cos \theta \quad \sec(\theta + 2\pi n) = \sec \theta\end{aligned}$
Quotient Identities	Double Angle Identities	Half Angle Identities
$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$	$\begin{aligned}\tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \sin(2\theta) &= 2 \sin \theta \cos \theta \\ &= \frac{2 \tan \theta}{1 + \tan^2 \theta} \\ &= (\cos \theta + \sin \theta)^2 - 1 \\ &= 1 - (\cos \theta - \sin \theta)^2 \\ \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta \\ &= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\end{aligned}$	$\begin{aligned}\sin\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ &\text{or} \\ \sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta) \\ \cos\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ &\text{or} \\ \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta) \\ \tan\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \\ &= \frac{1 - \cos \theta}{\sin \theta} \\ &= \frac{\sin \theta}{1 + \cos \theta} \\ &\text{or} \\ \tan^2 \theta &= \frac{1 - \cos 2\theta}{1 + \cos 2\theta}\end{aligned}$
Product to Sum Identities	Triple Angles	Sum and Difference Identities
$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)] \\ \cos \alpha \sin \beta &= \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]\end{aligned}$	$\begin{aligned}\sin(3\theta) &= 3 \sin \theta - 4 \sin^3 \theta \\ \cos(3\theta) &= 4 \cos^3 \theta - 3 \cos \theta \\ \tan(3\theta) &= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}\end{aligned}$	$\begin{aligned}\tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \\ \cot(\alpha \pm \beta) &= \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha} \\ \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta\end{aligned}$
Trigonometric Functions in terms of the other Ratios	Sum and Difference to Product Identities	Further Tangent Identities
$\begin{aligned}\tan \theta &= \pm \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta} = \pm \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} = \frac{1}{\cot \theta} = \pm \sqrt{\sec^2 \theta - 1} = \pm \frac{1}{\sqrt{\csc^2 \theta - 1}} \\ \sin \theta &= \pm \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}} = \pm \sqrt{1 - \cos^2 \theta} = \pm \frac{1}{\sqrt{1 + \cot^2 \theta}} = \pm \frac{\sqrt{\sec^2 \theta - 1}}{\sec \theta} = \frac{1}{\csc \theta} \\ \cos \theta &= \pm \frac{1}{\sqrt{1 + \tan^2 \theta}} = \pm \sqrt{1 - \sin^2 \theta} = \pm \frac{\cot \theta}{\sqrt{1 + \cot^2 \theta}} = \frac{1}{\sec \theta} = \pm \frac{\sqrt{\csc^2 \theta - 1}}{\csc \theta}\end{aligned}$	$\begin{aligned}\sin \alpha + \sin \beta &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \sin \alpha - \sin \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha + \cos \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha - \cos \beta &= -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)\end{aligned}$	$\begin{aligned}\tan(45^\circ - \theta) &= \frac{1 - \tan \theta}{1 + \tan \theta} \\ \tan(45^\circ + \theta) &= \frac{1 + \tan \theta}{1 - \tan \theta} \\ \tan\left(\frac{\pi}{4} - \theta\right) &= \frac{1 - \tan \theta}{1 + \tan \theta} \\ \tan\left(\frac{\pi}{4} + \theta\right) &= \frac{1 + \tan \theta}{1 - \tan \theta}\end{aligned}$

Trigonometry

Ratios and Laws

Trigonometric Ratios



$$0 < \alpha < 90^\circ$$

$$0 < \beta < 90^\circ$$

$$\tan \alpha = \frac{a}{b} \quad \tan \beta = \frac{b}{a}$$

$$\sin \alpha = \frac{a}{c} \quad \sin \beta = \frac{b}{c}$$

$$\cos \alpha = \frac{b}{c} \quad \cos \beta = \frac{a}{c}$$

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{b}{a} \quad \cot \beta = \frac{1}{\tan \beta} = \frac{a}{b}$$

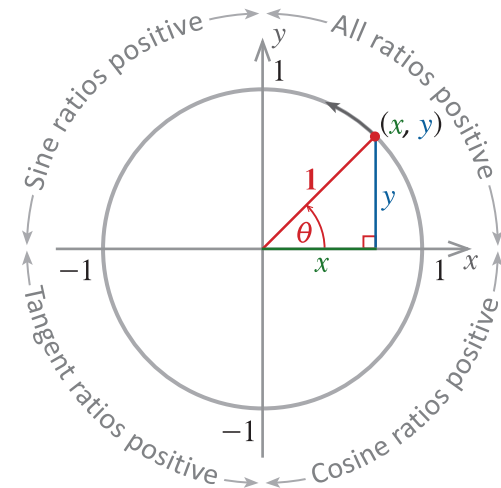
$$\csc \alpha = \frac{1}{\sin \alpha} = \frac{c}{a} \quad \csc \beta = \frac{1}{\sin \beta} = \frac{c}{b}$$

$$\sec \alpha = \frac{1}{\cos \alpha} = \frac{c}{b} \quad \sec \beta = \frac{1}{\cos \beta} = \frac{c}{a}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

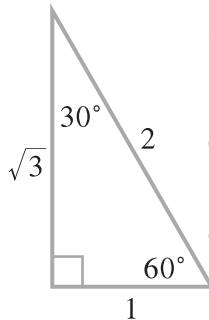
Unit Circle and Signs in Quadrants



$$\tan \theta = \frac{y}{x} \quad \sin \theta = \frac{y}{1} = y \quad \cos \theta = \frac{x}{1} = x$$

$$\cot \theta = \frac{x}{y} \quad \csc \theta = \frac{1}{y} \quad \sec \theta = \frac{1}{x}$$

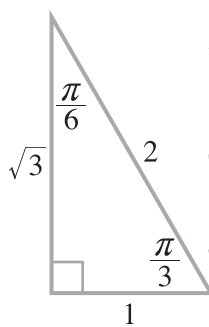
Exact Ratios



$$\tan 60^\circ = \sqrt{3} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$$

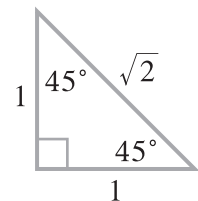
$$\cos 60^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$



$$\tan \frac{\pi}{3} = \sqrt{3} \quad \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \sin \frac{\pi}{6} = \frac{1}{2}$$

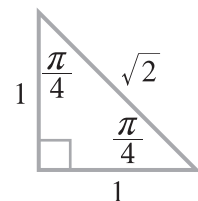
$$\cos \frac{\pi}{3} = \frac{1}{2} \quad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$



$$\tan 45^\circ = 1$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

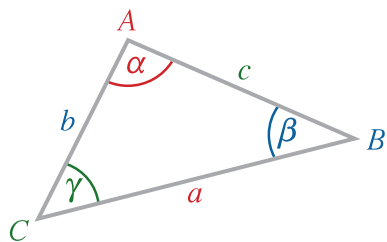


$$\tan \frac{\pi}{4} = 1$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Sine Law for Triangles



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\text{Area} = \frac{1}{2} ab \sin \gamma = \frac{1}{2} ac \sin \beta = \frac{1}{2} bc \sin \alpha$$

$$\alpha = \sin^{-1} \left(\frac{a \sin \beta}{b} \right) = \sin^{-1} \left(\frac{a \sin \gamma}{c} \right)$$

$$\beta = \sin^{-1} \left(\frac{b \sin \alpha}{a} \right) = \sin^{-1} \left(\frac{b \sin \gamma}{c} \right)$$

$$\gamma = \sin^{-1} \left(\frac{c \sin \alpha}{a} \right) = \sin^{-1} \left(\frac{c \sin \beta}{b} \right)$$

Cosine Law for Triangles

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \quad \alpha = \cos^{-1} \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \quad \gamma = \cos^{-1} \left(\frac{a^2 + b^2 - c^2}{2ab} \right)$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta \quad \beta = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2ac} \right)$$

Tangent Law for Triangles

$$\frac{a-b}{a+b} = \frac{\tan \left[\frac{1}{2} (\alpha - \beta) \right]}{\tan \left[\frac{1}{2} (\alpha + \beta) \right]}$$

$$\frac{b-c}{b+c} = \frac{\tan \left[\frac{1}{2} (\beta - \gamma) \right]}{\tan \left[\frac{1}{2} (\beta + \gamma) \right]}$$

$$\frac{a-c}{a+c} = \frac{\tan \left[\frac{1}{2} (\alpha - \gamma) \right]}{\tan \left[\frac{1}{2} (\alpha + \gamma) \right]}$$

Mollweide's Rule

$$\frac{a+b}{c} = \frac{\cos \left[\frac{\alpha - \beta}{2} \right]}{\sin \left[\frac{\gamma}{2} \right]} \quad \frac{a-b}{c} = \frac{\sin \left[\frac{\alpha - \beta}{2} \right]}{\cos \left[\frac{\gamma}{2} \right]}$$

Degrees and Radians

If θ is an angle in degrees and α is the same angle in radians:

$$\theta = \alpha \times \frac{180}{\pi}$$

$$\alpha = \theta \times \frac{\pi}{180}$$

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