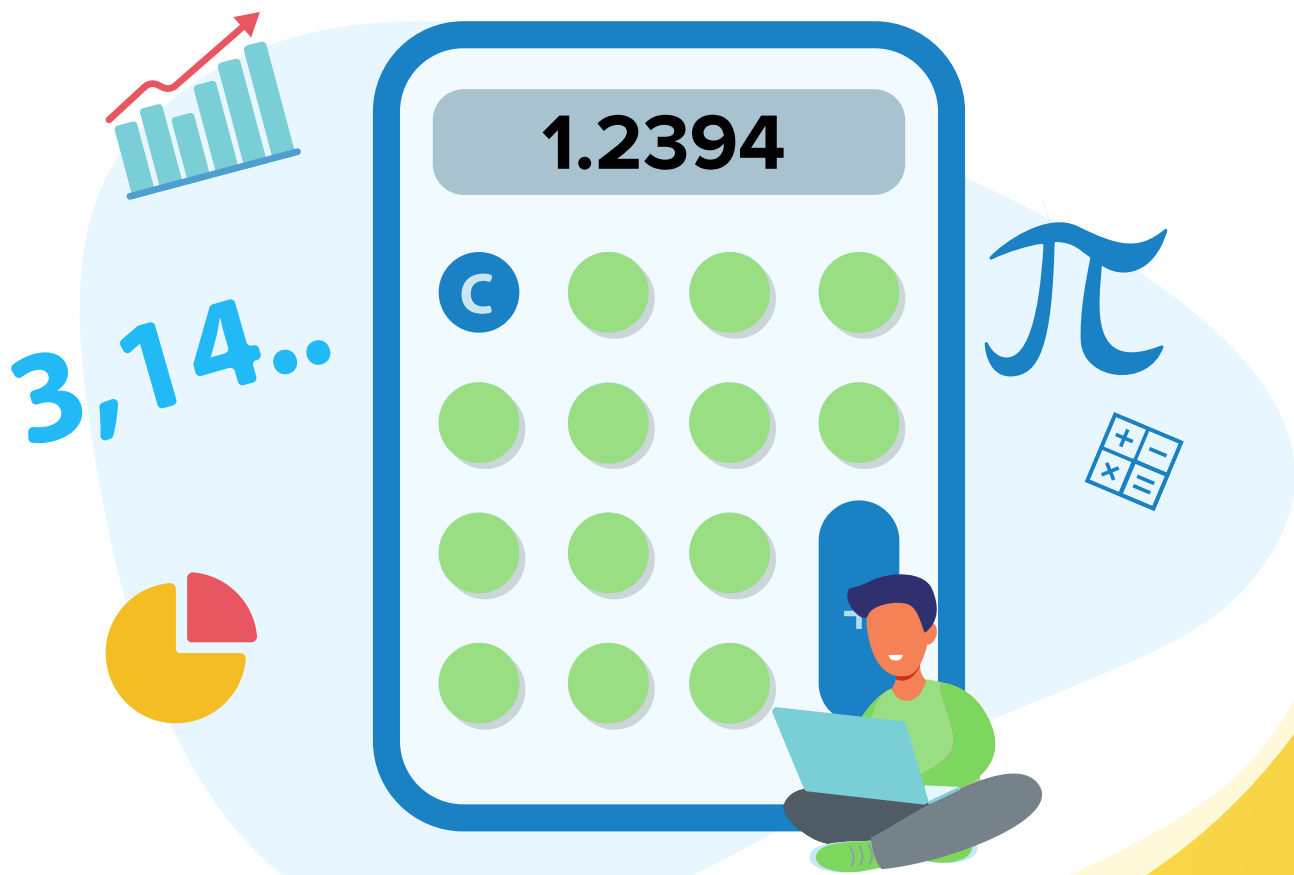


Mathletics

Decimal Worksheets



Student book

Year 9

DECIMALS

A decimal number is based on place value. 214.84 has 2 hundreds, 1 ten, 4 units, 8 tenths and 4 hundredths. Sometimes different 'levels' of place value are needed and a decimal number needs to be approximated. Sometimes decimals go on forever like 0.333333...



Answer these questions, *before* working through the chapter.

I used to think:

Is 23.4 nearer to 23 or 24?

What does it mean, if the ratio of peanuts to cashew nuts in a cake is 4:1?

What do the dots in $0.\dot{5}0\dot{9}$ mean?

Answer these questions, *after* working through the chapter.

But now I think:

Is 23.4 nearer to 23 or 24?

What does it mean, if the ratio of peanuts to cashew nuts in a cake is 4:1?

What do the dots in $0.\dot{5}0\dot{9}$ mean?



What do I know now that I didn't know before?

Decimals and Rounding

Is 21.4 closer to 21 or 22?

'Rounding' or 'rounding off' is a method of approximating a number's value. For example we may need a number accurate to the nearest unit, or nearest 10 or nearest thousandth, depending on different situations.

If the digit in the decimal place after the required accuracy is a 4 or less, then round down. If it is a 5 or more, then round up.

Rounding examples

- a Round 3.4 to the nearest unit.

Check this digit

The digit after the unit is a 4 so round down to the nearest unit. The nearest unit to 3.4 is 3

- b Round 3.7 to the nearest unit.

Check this digit

The digit after the unit is a 7 so round up to the nearest unit. The nearest unit to 3.7 is 4

- c Write 324.26 to the nearest hundred.

Check this digit

The digit after the hundred digit (3) is a 2 so round down to the nearest hundred. The nearest hundred to 324.26 is 300

- d Write 8345.276 to the nearest hundredth.

Check this digit

The digit after the hundredth digit (7) is a 6 so round up to the nearest hundredth. The nearest hundredth is 8345.24

- e Write 0.3687 correct to two decimal places.

Check this digit

The digit after the second decimal place is an 8 so round up. The number correct to two decimal places is 0.37

- f Write 0.6382 correct to one decimal place.

Check this digit

The digit after the first decimal place is a 3 so round down. The number correct to one decimal place is 0.6

- g Write 45.3857145 correct to four decimal places.

Check this digit

The digit after the fourth decimal place is a 1 so round down. The number correct to four decimal places is 45.3857

1. Round the following numbers to the nearest unit:

a 102.456

b 102.654

c 0.712

d 1999.98

2. Round as asked:

a Write 34.567 correct to 2 decimal places.

b Write 216.84 correct to 1 decimal place.

c Write 1935.3752 to the nearest hundred.

d Write 2048.4739 correct to the nearest unit.

e Write 0.0671 correct to 1 decimal place.

f Write 56.229 826 39 correct to 5 decimal places.

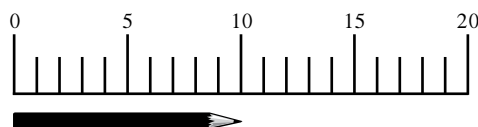
3. The length of a room is 5.52 m and the breadth is 3.41 m. Find the area correct to 2 decimal places.
(only round at the end)

Significant Figures

In decimal numbers, some of the digits are 'significant' and others are not. This depends on the accuracy of a measurement. Each number has a first significant figure and a last significant figure. 0.903 has first significant figure 9 and last significant figure 3. (The words 'digit' and 'figure' mean the same thing here.)

How to find the first significant figure

The first significant figure is the leftmost nonzero digit



The length of the pencil is 10 cm which could be written as 0.10 m or 0.000 10 km. In each measurement of the length of the pencil, the first significant figure is 1 because it is the first nonzero digit.

For example, 26.54 has first significant digit 2 and 0.843 has first significant digit 8.

Finding the last significant figure can be a bit tricky. There are three types of numbers we need to look out for when identifying the last significant digit.

Type 1: The last digit is nonzero.

If the last digit is nonzero, then the last digit is also the last significant digit. For example:

5008
 ↑
 Last digit is nonzero (8)

0.4007
 ↑
 Last digit is nonzero (7)

Both these numbers have first significant digit 5 and last significant digit 8.

Type 2: The last digit is a 0 and is after the decimal point

1234.0
 ↑
 The last digit is a zero after the decimal point

15.030
 ↑
 The last digit is a zero after the decimal point

Both these numbers have first significant digit 1 and last significant digit 0.

Type 3: A whole number ending with a 0 (no decimal point)

This is where it gets tricky. It depends on the accuracy of the measurement. Look at the number 200.

200
 {
 If it was rounded to the nearest hundred (for example, from 225 or 183) then only the 2 is accurate and it has only one significant figure (the 2).

200
 {
 If it was rounded to the nearest ten (for example from 199 or 202) then the first two digits are accurate and so there are two significant figures (the 2 and the first 0).

200
 {
 If it was rounded to the nearest unit then all three digits are significant.

Here are some examples.

Find the first and last significant figures of the following

a 158

The first significant figure is 1.

The last significant figure is 8 (Type 1).

b 308.0

The first significant figure is 3.

The last significant figure is 0 (Type 2).

c 3080 (after rounding to nearest 10)

The first significant figure is 3.

The last significant figure is 8 (Type 3).

c 25 000 (after rounding to the nearest 1000)

The first significant figure is 2.

The last significant figure is 5 (Type 3).

Counting Significant Figures

To find the number of significant figures, count the digits from the leftmost and rightmost significant numbers.

Find the first and last significant figures of the following

a 528

The first significant figure is 5.

The last significant figure is 8 (Type 1).

There are 3 significant figures.

b 23.070

The first significant figure is 2.

The last significant figure is the second 0 (Type 2).

There are 5 significant figures.

c 0.05201

The first significant figure is 5.

The last significant figure is 1 (Type 1).

There are 4 significant figures.

d 450.3010

The first significant figure is 4.

The last significant figure is the third 0 (Type 2).

There are 7 significant figures.

e 3200

(if this has been rounded to nearest hundred)

The first significant figure is 3.

The last significant figure is 2 (Type 3).

There are 2 significant figures.

f 3200

(if that has been rounded to the nearest 10)

The first significant figure is 3.

The last significant figure is the first 0 (Type 3).

There are 3 significant figures.

g 3200

(if this has been rounded to nearest unit)

The first significant figure is 3.

The last significant figure is the second 0 (Type 3).

There are 4 significant figures.

h 3200.0

The first significant figure is 3.

The last significant figure is the third 0 (Type 2).

There are 5 significant figures.

Rounding to Significant Figures

Sometimes rounding is done to significant figures instead of decimal places. The same method applies, the only difference is instead of checking the next digit according to decimal place, check the next digit according to **significant figures**.

If the digit after the required accuracy is a 4 or less, then round down. If it is a 5 or more, then round up.

Round the following numbers

- a Round 3.77 to 2 significant figures.

↑ ↑ ↑
1 2 3

3.77 to 2 significant figures is 3.8.

Not significant

- b Write 0.06542 correct to 2 significant figures.

↑ ↑ ↑ ↑ ↑
1 2 3 4

0.06542 to 2 significant figures is 0.065.

- c Write 7 563 528 correct to 3 significant figures.

↑ ↑ ↑ ↑ ↑ ↑ ↑
1 2 3 4 5 6 7

7 563 528 to 3 significant figures is 7 560 000.

- d Write 11.099 correct to 4 significant figures.

↑ ↑ ↑ ↑ ↑
1 2 3 4 5

11.10 correct to 4 significant figures.

↑
The zero on the end is necessary to make 4 significant figures

1. How many significant figures are in each of these numbers?

- a 342
 - b 342.00
 - c 0.4501
 - d 0.004 501
 - e 9200.0
 - f 0.0027
 - g 51302.067
 - h 0.002 700
-

2. Round these numbers:

- a Write 345.98 correct to 3 significant figures.
 - b Write 345.98 correct to 4 significant figures.
 - c Write 0.003 47 correct to 3 significant figures.
 - d Write 123 456 correct to 3 significant figures.
-

3. Find 2 possible numbers which can be written correct to 3 significant figures as:

- a 3210
- b 0.0452

Ratios

Ratios are easier to understand from an example than from their definition at first. So let's start with an example

In a class of boys and girls, there are 12 boys

- How many girls are in the class if the ratio of boys to girls is 1:2?
This means that for every 1 boy there are 2 girls. So if there are 12 boys then there are $12 \times 2 = 24$ girls.
- How many girls are in the class if the ratio of boys to girls is 2:1?
This means that for every 2 boys there is 1 girl. So in a class with 12 boys there are $12 \div 2 = 6$ girls.
- If the class has 18 girls, then what is the ratio of boys to girls?
There are 12 boys and 18 girls so the ratio of boys to girls is 12:18 which can be simplified to 2:3.
- If the ratio of boys to girls is 3:2 then how many students in total are in the class?
For every 3 boys there are 2 girls. This ratio can also be written as 12:8 (by multiplying both sides by 4). So if there are 12 boys it means there are 8 girls.
The total number of students is $12 + 8 = 20$.

Sometimes ratios can compare more than 2 things

In a cupboard the ratio of green socks to red socks to blue socks is 2:4:7.

- If there are 4 green socks, how many red and blue socks are there?
 - For every 2 green socks there are 4 red socks. If there are 4 (or 2×2) green socks there must be $2 \times 4 = 8$ red socks.
 - For every 2 green socks there are 7 blue socks. If there are 4 (or 2×2) green socks there must be $2 \times 7 = 14$ blue socks.
- If there are 12 red socks, how many green and blue socks are there?
 - For every 4 red socks there are 2 green socks. If there are 12 (or 3×4) green socks there must be $3 \times 2 = 6$ red socks.
 - For every 4 red socks there are 7 blue socks. If there are 12 (or 3×4) green socks there must be $3 \times 7 = 21$ blue socks.
- If there are 39 socks altogether, then how many green socks are in the cupboard?
According to the ratio for every 13 (or $2 + 4 + 7$) socks, 2 of them are green. This means $\frac{2}{13}$ of the socks are green.

$$\begin{aligned}\therefore \text{green socks} &= \frac{2}{13} \times \text{total socks} \\ &= \frac{2}{13} \times 39 = 6\end{aligned}$$

Ratios can be defined as comparing the number of like quantities relative to each other.

Rates

Rates compare unlike quantities. The most common is speed which compares distance to time using km/h or m/s. For example, if a car is driving at 20 km/h it means that after every hour it will travel a distance of 20 km.

While you're on a hike you notice you walk 2 metres each second. How long will it take you to walk a kilometre?

A kilometre is 1000 metres. If you walk at 2 m/s, to walk 1000 m it will take

$$\frac{1000 \text{ m}}{2 \text{ m/s}} = 500 \text{ s}$$

So, it will take 500 seconds for you to walk a kilometre which is 8 minutes and 20 seconds.

Sometimes it's important to know how to convert between the units of rates, like changing an amount from m/s to km/h.

A pipe fills a tank with water at a constant rate of 10.8 L/h (Litres per hour). Convert this to mL/s (millilitres per second).

Step 1: Write this as a fraction: $\frac{10.8 \text{ L}}{1 \text{ h}}$

Step 2: Convert the numerator and denominator as necessary:

$$= \frac{(10.8 \times 1000) \text{ mL}}{(1 \times 3600) \text{ s}}$$

Step 3: Simplify: $= \frac{10\,800 \text{ mL}}{3600 \text{ s}}$

$$= 3 \text{ mL/s}$$

How many kilometres does a driver driving at a constant speed of 30 m/s travel in 5 hours?

Step 1: Convert the speed to km/h:

$$\begin{aligned} & \frac{30 \text{ m}}{1 \text{ s}} \\ &= \frac{(30 \div 1000) \text{ km}}{(1 \div 3600) \text{ h}} = \frac{0.030 \text{ km}}{1 \text{ h}} \times \frac{3600}{1} \\ &= 108 \text{ km/h} \end{aligned}$$

Step 2: Use the new rate to find the number of metres travelled in 5 hours:

$$\begin{aligned} \text{kilometres travelled} &= 108 \text{ km/h} \times 5 \text{ h} \\ &= 540 \text{ km} \end{aligned}$$

1. Simplify these ratios as much as possible:

a 16:4

b 25:5

c 9:21

d 8:30

2. In a group of people there are 4 boys and 16 girls

a What is the ratio of boys to girls in simplest form?

b What is the ratio of girls to boys in simplest form?

3. In a cake recipe the ratio for the ingredients honey, flour and water is 3:10:6

a If a cake contains 30 parts of honey, how many parts of flour and water does it contain?

b If there are 57 parts of all these ingredients together, how many parts are flour?

c If 12 parts of water are used, how many parts in total make up all the ingredients of the cake?

4. £ 3600 is to be divided between Melissa and Garthe.

a Find how much money each gets if the ratio of Melissa's money to Garthe's money is 5:1

b Find how much money each gets if the ratio of Melissa's money to Garthe's money is 2:7

5. Let's say you read 434 pages in a week.

a Convert this to a daily rate?

b How long would it take you to read a series of books totalling 3255 pages?

6. Convert 18 km/h to m/s.

7. Convert 345 m/s to km/h.

8. A tank fills up at a rate of 300 mL each second at a constant rate:

- a Write this as a rate and then convert this rate to L/min.

- b Convert the rate to L/h.

- c How much liquid will be in the tank after 2 hours?

9. The scale of a map says 1:7500. How far are two points in real life, if on the page they are 10.2 cm apart?

10. Petrol costs 0.13 pence per millilitre.

- Convert this to a rate of pounds per litre.
- If the car uses 6.5 L for 100 km, how much petrol is needed to drive 300 km.
- How much will it cost to drive 300 km.

Recurring Decimals

A recurring decimal is a decimal number whose decimal digits repeat a sequence without ever stopping. For example 0.682 682 682 682... or 0.545 454 54... or 0.333 333 333... They're also called 'repeating decimals'. To write these easily, dots are placed over the digits that repeat to save writing really long decimals. Here are some examples.

- $0.333\ 333\ 3... = 0.\dot{3}$ (1 repeating digit)
- $0.653\ 535\ 3... = 0.6\dot{5}\dot{3}$ (2 repeating digits)
- $0.836\ 783\ 678\ 367... = 0.\dot{8}36\dot{7}$ (4 repeating digits)

Each recurring decimal can be written as a fraction $\frac{a}{b}$ where a and b are both integers. How is this fraction found?

Here are some examples

a $0.888\ 88...$

Let $x = 0.888\ 88...$

Multiply both sides by 10

$$\therefore 10x = 8.888\ 88...$$

$$\therefore 10x - x = 8.888\ 88... - 0.888\ 88...$$

$$\therefore 9x = 8$$

$$\therefore x = \frac{8}{9}$$

$$\therefore 0.888\ 88... = \frac{8}{9}$$

Check these values on a calculator

b $0.535\ 353...$

Let $x = 0.535\ 353...$

Multiply both sides by 100

$$\therefore 100x = 53.535\ 353...$$

$$\therefore 100x - x = 53.535\ 353... - 0.535\ 353...$$

$$\therefore 99x = 53$$

$$\therefore x = \frac{53}{99}$$

$$\therefore 0.535\ 353... = \frac{53}{99}$$

In the second step of **a** both sides we multiplied by 10, and in the second step of **b** both sides were multiplied by 100. When changing recurring decimals to fractions multiply both sides by 10^n where n is the amount of repeating digits.

There is a little extra trick when not all the digits are repeating digits.

Convert $0.4\dot{6}\dot{7}$ to a fraction

Let $x = 0.4\dot{6}\dot{7}$

Not all the decimal digits are repeating. There will be an extra trick

$$\therefore 100x = 46.767\ 676...$$

There are 2 repeating digits (6 and 7) so multiply both sides by $10^2 = 100$

$$\therefore 100x - x = 46.767\ 676... - 0.467\ 67...$$

$$\therefore 99x = 46.3$$

$$\therefore 990x = 463$$

Extra trick – multiply both sides to remove the decimal

$$\therefore x = \frac{463}{990}$$

$$\therefore 0.4\dot{6}\dot{7}... = \frac{463}{990}$$

1. Write these recurring decimals using dots on top of the digits:

a $0.222\dots$

b $0.868\,686\,86\dots$

c $0.120\,312\,031\,203\dots$

d $0.345\,645\,645\,6\dots$

2. Write these in repeating form:

a $0.\dot{4}$

b $0.3\dot{2}$

c $0.10\dot{9}$

d $7.3\dot{6}2\,4\dot{1}$

3. Convert these to fractions in their simplest form:

a $0.\dot{6}$

b $0.\dot{3}\dot{5}$

c $0.1\dot{2}\dot{3}$

d $2.\dot{7}\dot{8}$

4. Prove that $0.\dot{9} = 1$.

(Hint: Convert $0.\dot{9}$ into a fraction)

5. Convert these decimals to fractions:

(Hint: There is an extra trick in these questions)

a $0.4\dot{5}$

b $0.0\dot{2}1$

c $0.17\dot{3}$

d $0.524\dot{9}$

Mathletics



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