Probability

Curriculum Ready

Mathletics

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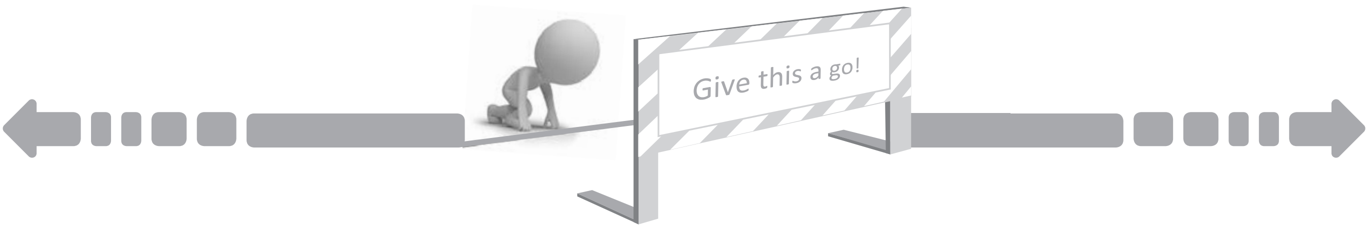
This booklet is all about the different ways probabilities can be presented and calculated.

Investigations into probability can have interesting results.
One famous investigation involves making a choice given three options.

A ball is hidden under one of three cups.

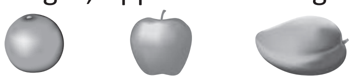
You choose which cup it is under and then get shown another cup it is NOT under.
Do you think it is better to stick with your original choice or change to the other remaining cup?

Try for yourself and see what conclusion you make.
For more information about this, search for the Monty Hall Problem.



Q A class of students were surveyed about whether or not they liked these three fruits:
Oranges, Apples and Mangoes.

Students who liked:



- Only one piece of fruit
  Oranges $= 8$
  Apples $= 4$
  Mangoes $= 7$
- Two pieces of fruit
  Oranges and Apples $= 8$
  Oranges and Mangoes $= 4$
  Apples and Mangoes $= 7$
- All three peices of fruit
  Oranges, Apples and Mangoes $= 8$

Come up with a way of grouping and displaying this information in a single diagram.

Use this diagram to calculate the probability that a surveyed student selected at random liked oranges.

Theoretical probability

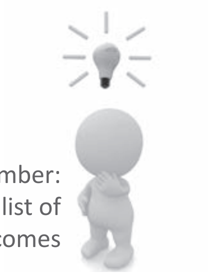
Theoretical probability is the expected chance of events written as fractions, decimals or percentages.

It compares how many times a particular event can happen with all the possible outcomes.

Theoretical probability of an event $P(E) = \dfrac{\text{Total number of favourable outcomes } n(E)}{\text{The total number of possible outcomes } n(S)}$

- $n(E) =$ the number of outcomes matching the result we are looking for.
- $n(S) =$ the total number of outcomes in the sample space.

Remember:
Sample space is a list of
all the possible outcomes



The total number of favourable outcomes can never be more than the total number of possible outcomes.

$\therefore$ The probability of an event can **only** be any value from $\mathbf{0}$ to $\mathbf{1}$.

$\therefore$ Probabilities in percentage form can **only** be any value from $\mathbf{0\%}$ to $\mathbf{100\%}$.

Here are some typical questions

(i) A pencil case has three blue, one green, two pink and two yellow highlighters.

α) List the sample space for all the possible outcomes if one highlighter was picked out at random.

The sample space $S = \{Blue, Blue, Blue, Green, Pink, Pink, Yellow, Yellow\}$

β) Calculate $P(Blue)$ after first calculating $n(Blue)$ and $n(S)$.

$$n(Blue) = 3 \qquad n(S) = 8$$

$$\therefore P(Blue) = \frac{n(Blue)}{n(S)} = \frac{3}{8} \quad \text{Probability as a fraction}$$

$$= 0.375 \quad \text{Probability as a decimal}$$

$$= 37.5\% \quad \text{Probability as a percentage}$$

(ii) If $P(E) = \dfrac{4}{5}$ and $n(S) = 15$, what is the value of $n(E)$?

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{5} \quad \text{Simplified fraction}$$

$$= \frac{n(E)}{15} \quad \text{Equivalent fraction with denominator of } 15$$

$$n(E) = 12 \quad \text{Numerator of simplified fraction} \times 3$$

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Theoretical probability

1 Write down the sample space S and calculate $n(S)$ for each of these spinners:

a

$S =$

$n(S) = n(Numbers) =$

b

$S =$

$n(S) = n(Colours) =$

c

$S =$

$n(S) = n(Options) =$

2 Calculate $P(E)$ accurate to 2 decimal places for these favourable outcomes and sample set values.

a $n(E) = 2, \quad n(S) = 25$

$P(E) =$

b $n(White\ flowers) = 10, \quad n(Flowers) = 14$

$P(White\ flowers) =$

c $n(Brown) = 7, \quad n(Cards) = 16$

$P(Brown\ card) =$

d $n(Odd\ numbers) = 12, \quad n(Numbers) = 33$

$P(Odd\ numbers) =$

3 Calculate $P(E)$, as a percentage of these:

a $n(E) = 1, \quad n(S) = 2$

$P(E) =$

b $n(E) = 3, \quad n(S) = 25$

$P(E) =$

c $n(E) = 36, \quad n(S) = 48$

$P(E) =$

d $n(E) = 5, \quad n(S) = 8$

$P(E) =$

4 Calculate $n(E)$ or $n(S)$ for each of these:

a $P(E) = \frac{1}{4}, \quad n(S) = 12$

$\therefore n(E) =$

b $P(E) = 30\%$ $\left(\text{i.e. } \frac{3}{10}\right), \quad n(E) = 15$

$\therefore n(S) =$

c $P(Orange) = 0.6$ $\left(\text{i.e. } \frac{6}{10}\right), \quad n(Oranges) = 21$

$\therefore n(Fruit) =$

d $n(Animals) = 12, \quad P(Duck) = 75\%$

$\therefore n(Ducks) =$

Theoretical probability

5 A bag contains the following wooden tiles with numbers or letters in the quantities given.

5 $\times 2$ **9** $\times 3$ **2** $\times 2$ **6** $\times 4$ **K** $\times 1$ **H** $\times 3$ **B** $\times 2$ **T** $\times 3$

A wooden tile is drawn at random from the bag.

a Write down the total values for each of these favourable outcomes:

(i) $n(9) = \boxed{}$ (ii) $n(B) = \boxed{}$ (iii) $n(Letter) = \boxed{}$ (iv) $n(Number) = \boxed{}$

(v) $n(H \text{ or } 6) = \boxed{}$ (vi) $n(Number < 9) = \boxed{}$ (vii) $n(7) = \boxed{}$ (viii) $n(Even) = \boxed{}$

b Write down the sample space for all the possible outcomes and the value of (S).

(i) $S =$

(ii) $n(S) = \boxed{}$

c Write these theoretical probabilities as simplified fractions if a tile is drawn at random from the bag.

(i) $P(9) = \dfrac{n(9)}{n(S)} = \dfrac{\boxed{}}{\boxed{}}$ (ii) $P(Even) = \dfrac{n(Even)}{n(S)} = \dfrac{\boxed{}}{\boxed{}}$ (iii) $P(B) = \dfrac{\boxed{}}{\boxed{}}$

(iv) $P(Number < 6) = \dfrac{\boxed{}}{\boxed{}}$ (v) $P(H \text{ or } B) = \dfrac{\boxed{}}{\boxed{}}$ (vi) $P(6 \text{ or } 9 \text{ or } T) = \dfrac{\boxed{}}{\boxed{}}$

6 A cup and ball guessing game is played with three cups and one ball. It is up to players to guess which cup has the ball underneath it after they have been shuffled around.

a Write down the sample space for all the possible outcomes when a cup is lifted up and the value of $n(S)$.

$S =$ $n(S) =$

b Calculate $P(No\ ball)$ and $P(Ball)$ when a cup is lifted up as a percentage.

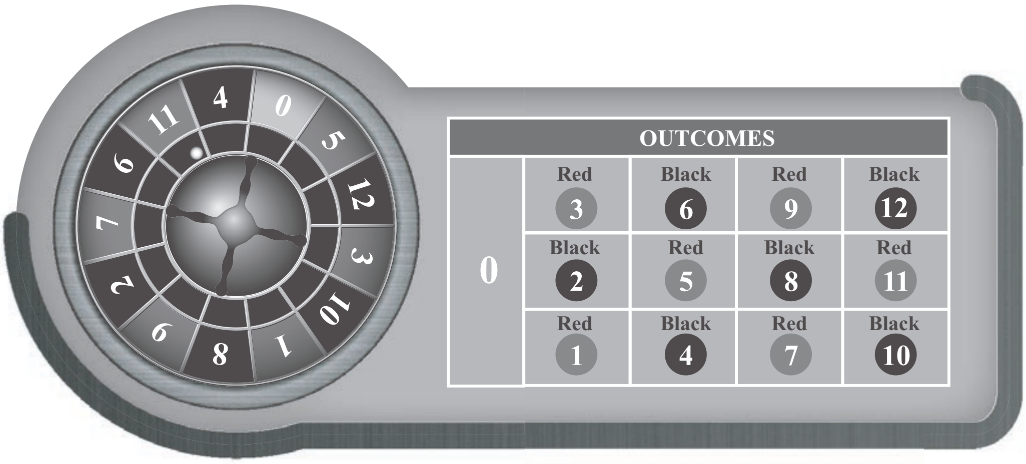
$P(No\ ball) =$ $P(Ball) =$

c Calculate $P(No\ ball) + P(Ball)$.

Theoretical probability

7 In Roulette a small white ball is spun on a moving wheel with red, black or green numbers.

People try to predict which number or colour the ball will stop on after each spin of the wheel.



a Write down the total values for each of these favourable outcomes:

(i) $n(Red\ numbers) =$ ☐ (ii) $n(Multiples\ of\ 3) =$ ☐ (iii) $n(Number < 9) =$ ☐

(iv) $n(Even\ numbers) =$ ☐ (v) $n(Single\ digit\ numbers) =$ ☐

b Write down the sample space for all the possible numbers and the value of (S).

(i) $S =$

(ii) $n(S) =$ ☐

c Calculate these theoretical probabilities as fractions then decimals rounded to 3 decimal places.

(i) $P(Black\ number) = \frac{\square}{\square}$

$= \square$ to 3 d.p.

(ii) $P(8) = \frac{\square}{\square}$

$= \square$ to 3 d.p.

(iii) $P(Prime\ number) = \frac{\square}{\square}$

$= \square$ to 3 d.p.

(iv) $P(Multiple\ of\ 5) = \frac{\square}{\square}$

$= \square$ to 3 d.p.

Theoretical probability

Words, fractions, decimals and percentages are all used to describe the **probability** of an outcome. This scale links the words used to describe chance to their **approximate** calculated probability value.

	highly unlikely	unlikely	equally likely (even chance)	likely	highly likely

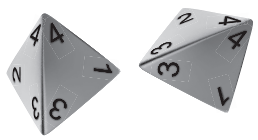
Fraction scale	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
Decimal scale	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Percentage scale	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

impossible	very unlikely	less than even chance	more than even chance	very likely	certain

Probability decreases ← → Probability increases

8 All the possible outcomes when two, four-sided dice are rolled are shown in the table below:

	First Die			
Second Die	1, 1	2, 1	3, 1	4, 1
	1, 2	2, 2	3, 2	4, 2
	1, 3	2, 3	3, 3	4, 3
	1, 4	2, 4	3, 4	4, 4



a Calculate $n(S)$ for the outcomes. $n(S) =$

b Calculate the number of these favourable outcomes:

(i) $n(Even\ sum) =$ (ii) $n(Sum\ of\ 5) =$

(iii) $n(Sum\ of\ 6) =$ (iv) $n(Sum\ of\ 3) =$

(v) $n(Sum < 5) =$ (vi) $n(At\ lease\ one\ 3) =$

(vii) $n(Sum > 2) =$ (viii) $n(Sum\ is\ a\ prime) =$

c Calculate and describe the probability of the favourable outcomes in part **b** occurring.

(i) $P(Even\ sum) =$

∴ ....

(ii) $P(Sum\ of\ 5) =$

∴ ....

(iii) $P(Sum\ of\ 6) =$

∴ ....

(iv) $P(Sum\ of\ 3) =$

∴ ....

(v) $P(Sum < 5) =$

∴ ....

(vi) $P(At\ lease\ one\ 3) =$

∴ ....

(vii) $P(Sum > 2) =$

∴ ....

(viii) $P(Sum\ is\ a\ prime) =$

∴ ....

Complementary events

So far we know that for the probability of an event:

- The total number of favourable outcomes can never be more than $n(S)$.
- The probability of an event $P(E)$ can **only** be any value from $\mathbf{0}$ to $\mathbf{1}$ or $\mathbf{0\%}$ to $\mathbf{100\%}$.

Complementary events in probability are about predicting the chance of the other possible events.

In other words, the probability of a certain event **not** happening.

$$P(Event\ \mathbf{NOT}\ Happening) = P(\overline{Event})$$

$$P(Winning\ a\ game) = P(\overline{Losing\ a\ game})$$

$P(\overline{E})$: The bar drawn over the event means complementary (or not)

Calculate these probabilities and look for a relationship between the two

Axis Airport had six jumbo jets, three airbuses and one helicopter land during a 1 hour period.
A plane spotter outside the airport was looking at what type of planes landed during this 1 hour period.

Calculate:

(i) $P(A\ jumbo\ jet\ was\ spotted) = \dfrac{Total\ number\ of\ jumbo\ jets\ that\ landed = 6}{Total\ number\ of\ aircraft\ that\ landed = 6 + 3 + 1 = 10}$

$= \dfrac{3}{5}$

(ii) $P(\overline{A\ jumbo\ jet\ was\ spotted}) = \dfrac{Total\ number\ of\ other\ aircraft\ landed = 3 + 1}{Total\ number\ of\ aircraft\ that\ landed = 6 + 3 + 1 = 10}$

$= \dfrac{2}{5}$

Can you see the relationship between the two probability calculations?

$$P(\overline{A\ jumbo\ jet\ was\ spotted}) = 1 - P(A\ jumbo\ jet\ was\ spotted)$$

$$= 1 - \frac{3}{5}$$

$$= \frac{2}{5}$$

This rule applies to all complementary probabilities:

- $P(\overline{E}) = 1 - P(E)$ when a decimal or fraction.
- $P(\overline{E}) = 100\% - P(E)\%$ when a percentage.

Complementary events

1 For each of these events, write down what the complementary event would be.

a $Event =$ turning a light switch on.

$\overline{Event} =$

b $Event =$ Sitting down.

$\overline{Event} =$

c $Event =$ Answering a question incorrectly.

$\overline{Event} =$

d $Event =$ Opening a jar.

$\overline{Event} =$

e $Event =$ Travelling by train.

$\overline{Event} =$

f $Event =$ Rolling an even number on a six-sided die.

$\overline{Event} =$

g $Event =$ Not hearing a sound.

$\overline{Event} =$

h $Event =$ Not understanding the task.

$\overline{Event} =$

2 Use probability notation to write the complementary probability for these in two different ways.

a $P(Fixed)$

$P(Broken)$ or $P(\overline{Fixed})$

b $P(Here)$

c $P(\overline{Healthy})$

d $P(\overline{Polite})$

e $P(Small)$

f $P(You\ saw\ something\ happen)$

3 Complete these calculations to find the complementary probability.

a $P(A\ blue\ card) = \frac{1}{4}$

$\therefore P(\overline{A\ blue\ card}) = \boxed{} - \boxed{}$

$= \boxed{}$

b $P(Winning) = 65\%$

$\therefore P(\boxed{}) = \boxed{} - \boxed{}$

$= \boxed{}$

Complementary events

4 Calculate the complementary probability for each of these:

a $P(Blue) = \frac{1}{3}$

$\therefore P(Not\ blue) =$

b $P(Good\ reception) = 85\%$

$\therefore P(Poor\ reception) =$

c $P(Arriving\ on\ time) = 0.30$

$\therefore P(\overline{Arriving\ on\ time}) =$

d $P(Raining\ tomorrow) = \frac{2}{5}$

$\therefore P(\overline{Raining\ tomorrow}) =$

e $P(\overline{Parrot\ talking}) = 0.17$

$\therefore P(Parrot\ talking) =$

f $P(\overline{Have\ green\ eyes}) = 74.4\%$

$\therefore P(Have\ green\ eyes) =$

5 A lucky dip with 50 vouchers contains these possible prizes:

- $30 \times \$5$ mobile phone credit
- $15 \times \$10$ mobile phone credit
- $4 \times \$25$ mobile phone credit
- $1 \times$ new prepaid mobile phone

Write these prize probabilities as simplified fractions if one voucher is chosen randomly.

a $P(\$25\ mobile\ phone\ credit)$

b $P(\overline{\$25\ mobile\ phone\ credit})$

c $P(New\ prepaid\ mobile\ phone)$

d $P(\overline{New\ prepaid\ mobile\ phone})$

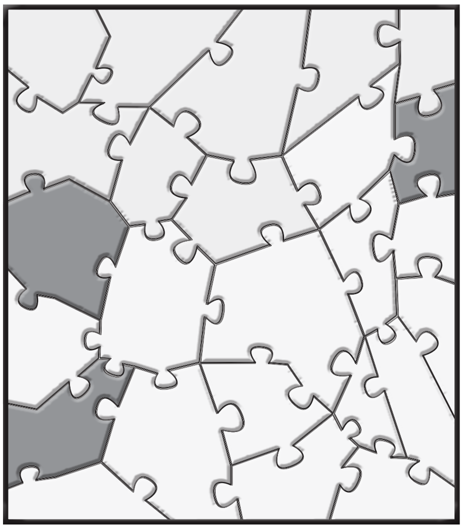
e $P(\$5\ or\ \$10\ mobile\ phone\ credit)$

f $P(\overline{\$25\ credit\ or\ a\ new\ prepaid\ mobile\ phone})$

g Comment on the relationship between the events for parts **e** and **f**.

Complementary events

6 The pieces in the completed jigsaw puzzle shown below get jumbled up. A piece is then taken from the jumbled pile at random. Calculate these probabilities as percentages for the selected puzzle piece:



a $P(Shaded\ puzzle\ piece)$

b $P(\overline{Corner\ puzzle\ piece})$

c $P(Edge\ puzzle\ piece)$

d $P(\overline{Edge\ puzzle\ piece})$

e $P(\overline{Puzzle\ piece\ that\ touches\ a\ shaded\ piece\ once\ solved})$

7 You are one in a group of 8 children at a birthday party playing ‘pass the parcel’. In this game, children sit in a circle and pass around a wrapped parcel until the music stops. Whoever is holding the parcel after the music stops unwraps one layer of wrapping to see if they have won a small prize. The child who unwraps the last layer of paper wins the main prize. Each child takes 2 seconds to pass the parcel to the next person.

a If the parcel starts with you, how many times will you hold it after 80 seconds of music (not including at the start of the game)?

b Not including at the start, calculate $P(\overline{Holding\ the\ parcel})$ during the first 80 seconds.

c Calculate $P(Winning\ the\ main\ prize)$ written as a decimal.

Independent and dependent events

When two or more simple events take place we call it a compound event.

There are two main types of compound events:

- **Independent:** The outcomes of one event does not affect the outcomes of the other.

  Eg: Flipping two coins.
  The outcome of one coin has no effect on the outcome of the other.

- **Dependent:** The outcome of one event affects the outcome of the other.

  Eg: Popping the balloon in a bunch that contains a prize.
  The chance of popping the winning balloon increases after each attempt as there are fewer balloons left to pop.

Random selections of two or more objects can happen one of two ways:

- **With replacement:** Each object selected is replaced before making the next selection. The sample space size is the same for all selections, so the outcome of each event is **independent** of the one before.

- **Without replacement:** Each object selected is not replaced before making the next selection. The sample space is smaller for the next selection, so the outcome of each event is **dependent** on what is left after the previous event.

Identify each of these types of events

(i) Tossing a coin and rolling a die (singular of dice).

Independent — Outcome of the coin toss does not affect the outcome on the die

(ii) Selecting two tiles together of the same colour from a bag.

Dependent — Two tiles together is a without replacement selection

(iii) Rolling a pair of normal playing dice.

Independent — Outcome of each die is not affected by the outcome on the other

(iv) An mp3 player selecting two songs by the same artist, one after the other while on random shuffle mode.

Dependent — There are less songs to randomly select from after the first one is played

Independent and dependent events

1. Identify each of these as dependent or independent events by ticking the correct term.

   a. Flipping two coins — ☐ Independent ☐ Dependent

   b. Flicking a number spinner and selecting a numbered card at random from a pack. — ☐ Independent ☐ Dependent

   c. Selecting two blunt pencils from a pencil case at the same time. — ☐ Independent ☐ Dependent

   d. Picking two out of three cups (one after the other) to see which one contains a hidden ball. — ☐ Independent ☐ Dependent

   e. Selecting two green marbles if the first marble was returned to the bag before selecting the second one. — ☐ Independent ☐ Dependent

   f. Guessing correctly the first two numbers to be drawn in a game of bingo. — ☐ Independent ☐ Dependent

   g. Randomly selecting seven tiles in a word game, then replacing and selecting another seven tiles. — ☐ Independent ☐ Dependent

   h. Two different people opening their books to the exact same page as each other. — ☐ Independent ☐ Dependent

   i. Two sheep giving birth to lambs on the same day. — ☐ Independent ☐ Dependent

   j. Guessing who will finish in the first two places of a race. — ☐ Independent ☐ Dependent

2. Bag 1 contains ten cards and each card has a different number (from 0 through to 9) written on it.
   Bag 2 contains five yellow cubes and five green cubes.
   Describe two independent events and two dependent events that can be explored through random selections from one or both of these bags.

   a. Independent events 1:

   b. Independent events 2:

   c. Dependent events 1:

   d. Dependent events 2:

Independent and dependent events

3 (i) Identify each of these compound events as either dependent or independent.
(ii) Describe how you could change each event into the other type.

a Rolling a die twice and recording the sum.

(i) ☑ Independent ☐ Dependent

(ii) Roll the die twice to record the sum, only if an odd number occurs on the first roll. The second role (and sum) is now dependent on the outcome of the first roll.

b Picking two coloured discs from a bag containing yellow, green and red discs without replacement.

(i) ☐ Independent ☐ Dependent

(ii)

c Guessing the number between 1 and 20 that Vaneeta is thinking of in two or more attempts.

(i) ☐ Independent ☐ Dependent

(ii)

d Recording the colour this spinner stops on each time it is spun and the number rolled on a four sided die.

(i) ☐ Independent ☐ Dependent

(ii)

e Selecting three numbers from a bag at random in descending order with replacement.

(i) ☐ Independent ☐ Dependent

(ii)

f Selecting one key from each of two identical sets of key that will open the same lock.

(i) ☐ Independent ☐ Dependent

(ii)

Mutually exclusive and inclusive events

Mutually exclusive events cannot happen at the same time.

Inclusive events can happen at the same time.

When rolling a die:

- A number that is odd or is a multiple of two cannot happen at the same time.
  ∴ these are mutually exclusive events
- A number that is odd or is a multiple of three can happen at the same time.
  ∴ these are inclusive events (or not mutually exclusive)

Both types of events use the words 'and', 'or', 'either' and 'at least' in probability statements.

- Inclusive and: Where events X and Y can happen.

  Eg: A musician playing the guitar (X) while singing (Y).

- Exclusive-Or: Where **either** event X **or** Y can happen but not at the same time.

  Eg: A person shouting (X) or whispering (Y).

- Inclusive-Or : Where events X or Y or both X and Y can happen.

  Eg: Jenny shaking hands (X) or Linda shaking hands (Y).
  (X **and** Y is Jenny and Linda shaking hands with each other or other people).

'At least' is used for inclusive-or statements. Because 'at least' means **either** X **or** Y **or** both X **and** Y:
'At least **either** events X **or** Y **or** both occurring'.

Write the type of exclusive and inclusive events each of these statements represent

(i) Picking one disc which is either blue or red from a bag containing red, blue and green discs.

Mutually Exclusive (Exclusive-Or): Blue or Red (but not both) colours can be selected.

(ii) Picking two discs, one blue and the other red, from a bag containing 3 red and 3 blue discs.

Inclusive-and: Picking a disc of each colour can happen at the same time.

(iii) Catching at least one of the 10 tadpoles in a pond using a net.

Inclusive-Or: One, two, three or more can be caught. A minimum of one must be caught.

(iv) An outcome of only one Tail when flipping three coins.

Mutually Exclusive (Exclusive-Or): Only coin 1, coin 2 or coin 3 can be a Tail, not a combination of this.

Mutually exclusive and inclusive events

1 Decide if these are mutually exclusive or inclusive events by ticking the right term.

a) Flipping a Head or Tail on two different coins.

☐ Mutually exclusive ☐ Inclusive

b) A light switch in the 'on' or 'off' position.

☐ Mutually exclusive ☐ Inclusive

c) Winning first or second prize in a local raffle with one ticket.

☐ Mutually exclusive ☐ Inclusive

d) Winning first or second prize in a local raffle with two tickets.

☐ Mutually exclusive ☐ Inclusive

2 A bag contains ten cards and each card has a different number (from 0 through to 9) written on it. Describe mutually exclusive and inclusive events that involve randomly selecting a card from this bag.

a) Mutually exclusive events:

b) Inclusive events:

3 Two boxes contain the following:

Box A: 1 orange, 1 blue and 5 yellow marbles.
Box B: 1 yellow, 1 green and 3 black marbles.

Describe two different a) mutually exclusive and b) inclusive events that involve randomly selecting two marbles (with or without replacement) from the same box, or one from each box contain the following:

a) Mutually exclusive events:

b) Inclusive events:

Mutually exclusive and inclusive events

4 Tick the correct type of exclusive or inclusive events each of these statements represent.

a A student selected from the class has either brown hair or brown eyes.

Exclusive Or | Inclusive Or | Inclusive And

b Dropping a cup and spilling all the contents.

Exclusive Or | Inclusive Or | Inclusive And

c One of two teachers selected randomly in a school catches public transport to school.

Exclusive Or | Inclusive Or | Inclusive And

d Boiling and freezing a container of water.

Exclusive Or | Inclusive Or | Inclusive And

e A person selected at random is either sitting down or standing up.

Exclusive Or | Inclusive Or | Inclusive And

f Rolling a number larger than 5 and an even number on a normal 6-sided die.

Exclusive Or | Inclusive Or | Inclusive And

g Spelling a word correctly and using it properly in a sentence.

Exclusive Or | Inclusive Or | Inclusive And

h Selecting a red card and the number 7 from a normal pack of playing cards

Exclusive Or | Inclusive Or | Inclusive And

i A student selected randomly during period 3 was doing Physical Education or Music.

Exclusive Or | Inclusive Or | Inclusive And

5 Earn yourself and AWESOME passport stamp with this one.
Professor Probability visits one day and during a chat exclaims:

"There is no such thing as a single inclusive, dependent event!"

Explain why you agree or disagree with the statement and give an example to support your answer.

Two-way tables

These are a great way to display all the pairs of outcomes for two events, actions or questions.

For example: A group of students where asked if they had ever gone surfing or kayaking before.

There are four different possible responses (outcomes) each student can give.

Yes to surfing and yes to kayaking
Yes to surfing and no to kayaking
No to surfing and yes to kayaking
No to surfing and no to kayaking

We can represent these outcomes in a two-way table.

		Kayaking	
		Yes	No
Surfing	Yes	← Yes ↑ Yes Students here have done both activities	← Yes ↑ No Students here only have been surfing
	No	← No ↑ Yes Students here have only been kayaking	← No ↑ No Students here have done neither activity

Each cell showing the outcome pairings is filled with the frequency of that outcome like this:

		Kayaking	
		Yes	No
Surfing	Yes	8	6
	No	4	12

The total number of students surveyed is the sum of all the values in the two-way table $(= 30)$.

Numbered red and black cards were selected at random and the outcomes recorded

		Number	
		Odd	Even
Color	Black	11	16
	Red	14	9

(i) How many cards were selected?

Number of cards selected $= 11 + 16 + 14 + 9$
$= 50$

(ii) How many red cards were selected with an odd number on them?

$n(Red, Odd) = 14$

(iii) How many black cards were selected?

$n(Black) = 11 + 16$
$= 27$

(iv) Which outcome had the highest frequency?

A black card with an even number;
$n(Black, Even) = 16$

Two-way tables

1 Complete the two-way tables for each of these:

a People were asked if they preferred their apple pie hot/cold and with/without ice cream.

		Ice Cream	
Apple pie			

b A decision spinner (Yes/No) spun either clockwise or counter-clockwise.

		Direction	

c A new local sports team asked its players for their preferred choice of team colours between black/white shorts and yellow, red or orange shirts.

2 Answer the questions for the two-way table below showing the results of a musical survey.

		Play guitar	
		Yes	No
Play flute	Yes	5	12
	No	14	23

a How many people were surveyed?

b How many people surveyed play both instruments?

c How many people surveyed play the flute?

d How many people surveyed can play one instrument only?

e $n(Play\ guitar, Do\ not\ play\ flute) =$

f Another 5 people who do not play either instrument are surveyed. What change needs to be made to the table?

Two-way tables

3 Aruma School has 200 students who travel from four surrounding towns. The towns where all the boys and girls in this school live was recorded in a two-way table.

	Girls	Boys
Neeuk Creek	18	21
Nooroon Plains	37	26
Dilkara	15	
Alba	42	33

a How many boys travel from Dilkara to attend the school?

b How many girls attend Aruma School?

c A large family with two sons and four daughters who attend Aruma School move from Nooroon Plains to Neeuk Creek. Complete the two-way table with the new values below.

	Girls	Boys
Neeuk Creek		
Nooroon Plains		

4 A group of people took part in a blind tasting test as part of a market research.
Each person had to say whether brand A or brand B tasted better. Then they needed to decide if the brands were better on their own, or with ingredient X or ingredient Y added.
Use the information collected to fill in the two-way table.

- 10 people preferred brand B without any added ingredients.
- Four times as many people preferred brand B with ingredient Y added, than on its own.
- Forty seven people preferred the product after ingredient Y was added.
- Thirty four people preferred the brands unchanged.
- An equal amount of people liked Brand A with nothing added and with ingredient X added.
- 120 people participated.

	Nothing Added	Ingredient	Ingredient
Brand:			
Brand:			

Two-way table probabilities

The fraction of observed results in any experiment/collection of trials is called the relative frequency.

Number of times it happens

$$\text{Relative frequency} = \frac{\text{The frequency of the outcome being observed}}{\text{The number of trials completed}}$$

The values along with the row and column totals make relative frequency calculations easy.

Complete the two-way table and use it to answer the given questions below

After tossing a coin and rolling a four-sided die together 100 times, the tally of each outcome pairing was recorded.

$(H, 1) =$ 卌 卌 ||| $(H, 2) =$ 卌 ||| $(H, 3) =$ 卌 卌 || $(H, 4) =$ 卌 卌 ||||

$(T, 1) =$ 卌 卌 卌 $(T, 2) =$ 卌 卌 |||| $(T, 3) =$ 卌 ||| $(T, 4) =$ 卌 卌 卌 |

(i) Record the observed results into a two-way table.

		4 sided die				
		1	2	3	4	**Total**
Coin	Hand (H)	13	8	12	14	47
	Tail (T)	15	14	8	16	53
	Total	28	22	20	30	100
		Total 1s	Total 2s	Total 3s	Total 4s	

Total heads

Total tails

Sum of the column/row totals should both equal the total number of trials.

(ii) Calculate the relative frequency and theoretical probability for the outcome $(T, 4)$.

Relative Frequency (or Experimental Probability)

$$n(T, 4) = 16, \quad n(Trials) = 100$$

Relative frequency of $(T, 4) = \dfrac{n(T,4)}{n(Trials)} = \dfrac{16}{100} = \dfrac{4}{25}$

Theoretical Probability

$$n(T, 4) = 1, \quad n(S) = 8$$

$$P(T, 4) = \frac{n(T,4)}{n(S)} = \frac{1}{8}$$

The more trials completed, the closer we expect the relative frequency to match the theoretical probability.

(iii) Calculate the relative frequency for flipping a Head when the number 3 was rolled.

$$n(H, 3) = 12, \quad n(Trials\ in\ which\ a\ 3\ was\ rolled) = 20$$

Relative frequency of heads when a 3 is rolled $= \dfrac{n(H,3)}{n(Trials\ in\ which\ a\ 3\ was\ rolled)}$

$= \dfrac{12}{20}$

$= \dfrac{3}{5}$ Simplified form

$= 60\%$ Percentage form

Two-way table probabilities

1 Complete these two-way tables using the collected data given.

a The position of two light switches observed 20 times.

(*Switch* 1, *Switch* 2)

(*On*, *On*): ||||
(*On*, *Off*): 𝍸
(*Off*, *On*): |||
(*Off*, *Off*): 𝍸 |||

		Switch 2		
		On	Off	Total
Switch 1	On			
	Off			
	Total			

b The results of flipping two coins 50 times.

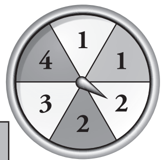
(*Coin* 1, *Coin* 2)

(H, H): 𝍸 |||
(H, T): 𝍸 𝍸 ||||
(T, T): 𝍸 𝍸 𝍸 ||
(T, H): 𝍸 𝍸 |

		Coin 2		
		Head (H)	Tail (T)	Total
Coin 1	Head (H)			
	Tail (T)			
	Total			

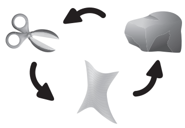
2 **a** Fill in the missing values for these two-way tables:

(i) Shades and numbers spinner.



	White	Grey	Total
1	18	15	
2		11	30
3	13		22
Total		35	

(ii) Scissors, Paper, Rock games.



		Player 2			
		Scissors (S)	Paper (P)	Rock (R)	Total
Player 1	Scissors (S)	8	9	5	
	Paper (P)	11		7	23
	Rock (R)	4	6		15
	Total		20		60

b Use the completed tables to answer these:

(i) How many spins were observed on the shades and numbers spinner?

(ii) How many games of Scissors, Paper, Rock were recorded?

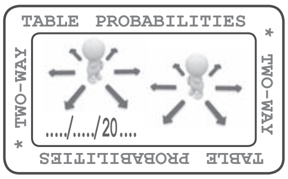
(iii) How many times did the spinner stop on a grey sector?

(iv) How many times did player 1 say ‘paper’?

(v) How many times did a game have scissors and paper (by either player) as the result?

(vi) If rock beats scissors, which player won the most games when this outcome occurred?

(vii) How many times did Player 1 and Player 2 make the same symbol?



Two-way table probabilities

3 This two-way table shows the results of a random survey of students in a school who were asked a yes/no question followed by a multiple choice question.

		Q2				
		A	B	C	D	**Total**
Q1	Yes (Y)	4	7	1	0	12
	No (N)	8	17	10	3	38
	Total	12	24	11	3	50

a. How many students were surveyed in this school?

b. How many students surveyed selected 'C' for Question 2?

c. What was the most common outcome for the two questions asked in this survey?

d. What outcome did not occur for the two questions asked in this survey?

e. What is the frequency for the outcome 'Yes, A'?

f. What is the relative frequency for the outcome 'Yes, A'?

g. What is the relative frequency for an answer of 'No' to Q1 as a percentage?

4 Numbers 1 through to 20 were printed on two packs of twenty cards. One pack printed using red ink (R) and the other printed using green ink (G). The two packs were then shuffled together.

a. Twenty four cards were randomly selected (with replacement) and the outcomes recorded. Complete the two-way table given using the recorded outcomes below.

$(G, 15)$ $(G, 1)$ $(G, 11)$ $(R, 6)$
$(R, 5)$ $(R, 18)$ $(R, 3)$ $(G, 17)$
$(R, 12)$ $(G, 8)$ $(R, 19)$ $(R, 2)$
$(G, 7)$ $(R, 1)$ $(G, 3)$ $(G, 6)$
$(G, 10)$ $(R, 3)$ $(G, 5)$ $(G, 6)$
$(R, 14)$ $(R, 16)$ $(G, 13)$ $(R, 5)$

		Colour		
		Red	Green	**Total**
Number	≤ 10			
	>10			
	Total			

b. Calculate the expected (theoretical) probability of selecting a green card with a number ≤ 10 and its relative frequency following the random selections.

c. If the number of random selections is increased greatly, what do you expect will happen to the theoretical probability and relative frequency values?

Two-way table probabilities

5 Children who successfully tossed a ring around a particular bottle at a local fair where then given the chance to pick a prize randomly from the lucky dip box.
The lucky dip box contained two different prizes: A mini mp3 player or a packet of bubble gum.
After one day at the fair, the results in the two-way table below were observed.

		Lucky Dip		
		Mp3	Gum	**Total**
Ring toss successful	Yes (Y)	2	48	50
	No (N)	0	0	0
	Total	2	48	50

a Explain why the values for 'No' are all zero.

b Write the relative frequency of each lucky dip prize in percentage form.

c Explain why you think there is a large difference between the two values in part b.

d After another 30 successful ring tosses, all the prizes drawn from the lucky dip were bubble gum. What is the new relative frequency for each lucky dip prize following the extra 25 wins?

6 Earn an Awesome passport stamp for this one.
Use the information given here to complete the two-way table below

- Relative frequency of the outcome $(Even, Up) = \frac{32}{115}$
- 64 Odd numbers were observed
- Relative frequency of the direction being 'Up' when an odd number was observed $= 25\%$

		Direction		
		Up	Down	**Total**
Number	Even			
	Odd			
	Total			

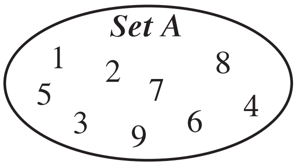
Set diagram basics

Venn diagrams show the members of collected data arranged into groups called sets.
They show what members are unique to a set and which ones occur in more than one set.

All data – including anything outside of the sets – are members of the Universal Set (U).

For example:

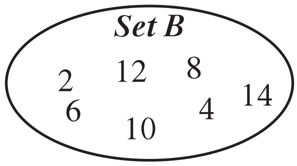
Set A: All intergers from 1 to 9



$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ Set notation

$n(A) = 9$ Number of members

Set B: All even integers between 1 and 15

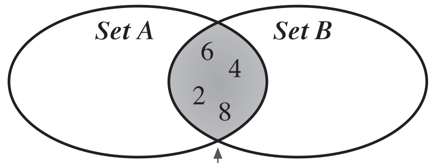


$A = \{2, 4, 6, 8, 10, 12, 14\}$ Set notation

$n(B) = 7$ Number of members

Intersection ($\cap$)

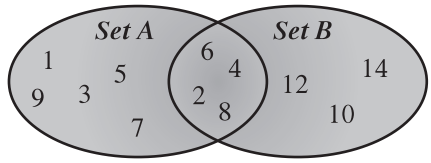
- Where Sets A **and** B overlap.
- So, $A \cap B = \{2, 4, 6, 8\}$ and $n(A \cap B) = 4$.



These are both integers between 1 and 9 AND even integers between 1 and 15

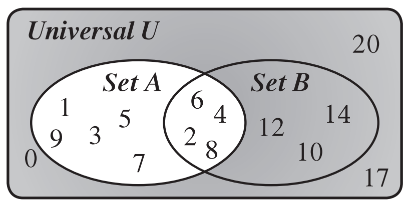
Union ($\cup$)

- The members that are in either set A **or** set B **or** both.
- So, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14\}$ and $n(A \cup B) = 12$.



Complement ($\overline{A}$)

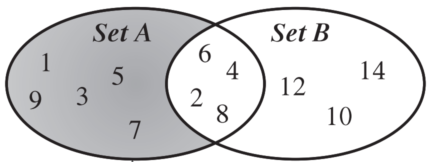
- Everthing that is **not** in the set.
- So, $\overline{A} = \{0, 10, 12, 14, 17, 20\}$ and $n(\overline{A}) = 6$.
  Also $\overline{B} = \{0, 1, 3, 5, 7, 9, 17, 20\}$ and $n(\overline{B}) = 8$.
- $A^c, \overline{A}, A', \tilde{A}, A^{\sim}$ are all different ways used to show the complement of set A.



Psst: Three numbers (0, 17, 20) were added to the Universal set that don't fit into sets A or B.

Difference ($-$)

- The difference (also called the relative complement) is used to find what is unique to a set.

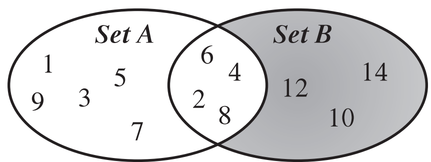


$A - B$ means Set A but **not** Set B

$\therefore A - B = \{1, 3, 5, 7, 9\}$

$n(A - B) = 5$.

The order is important



$B - A$ means Set B but **not** Set A

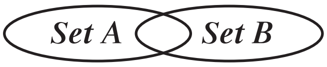
$\therefore B - A = \{10, 12, 14\}$

$n(B - A) = 3$.

Set diagram basics

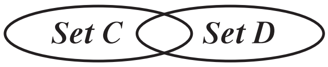
1 (i) Shade each of these diagrams to match the statements given.
(ii) Write the shaded area using the symbols ∩, ∪, – or $\overline{Set}$.

a (i) All the data in both sets



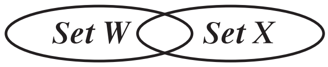
(ii)

b (i) All members in Set C



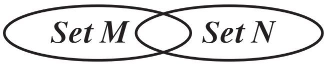
(ii)

c (i) Data is shared by both sets



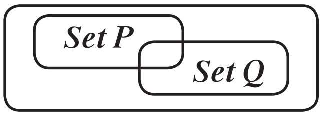
(ii)

d (i) Set N but not Set M



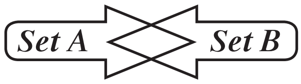
(ii)

e (i) Everything except Set Q



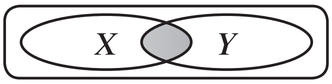
(ii)

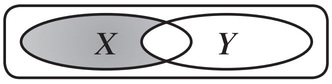
f (i) The members of Set B that are not unique to Set B

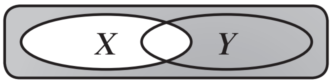


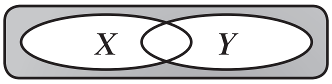
(ii)

2 Match these Venn diagrams with the correct description in the middle.

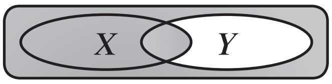


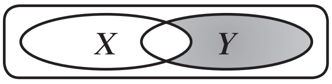


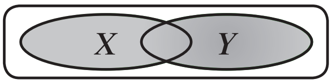


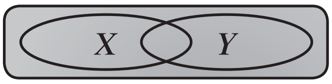


- X but not Y
- Y but not X
- X or Y or both
- X and Y
- neither X or Y
- not X
- not Y
- all data









3 A box contains ten balls, all numbered from 1 to 10.

Set A = balls containing multiples of 3, Set B = balls containing odd numbers.

a Write in the members of each set below:

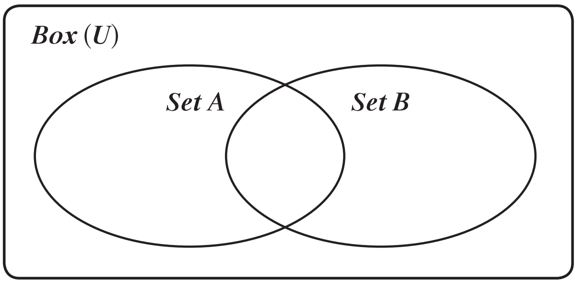
(i) $A = \{$ 3, 6, 9 $\}$

(ii) $B = \{\quad\}$

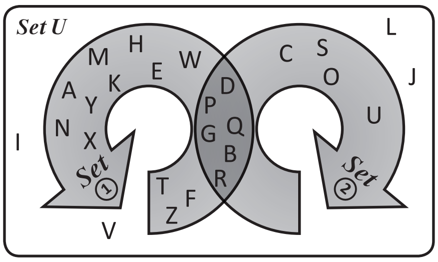
(iii) $U = \{\quad\}$

(iv) $A \cap B = \{\quad\}$

b Put the data into this Venn diagram.



4 Write down the members of the following sets displayed in the Venn diagram below:



(i) ① =

(ii) ① ∩ ② =

(iii) $\overline{②}$ =

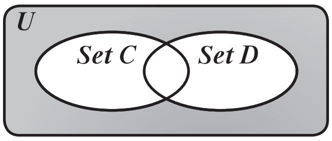
(iv) ② – ① =

(v) ① – ② =

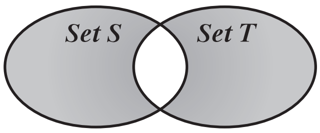
Set diagram basics

Symbols can be combined to represent other groupings of data

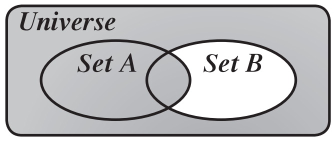
(i) $\overline{C \cup D}$
Everything outside of Sets C **or** D.



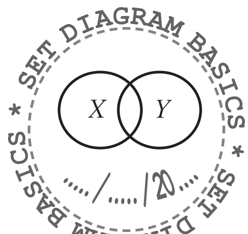
(ii) $\overline{S \cap T}$
Where Sets S **and** T do not overlap



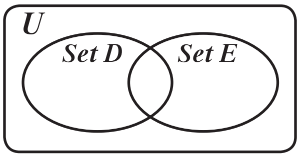
(iii) $\overline{B - A}$
Everything that is not unique to Set B



5 Shade each of these diagrams to match the information given underneath.

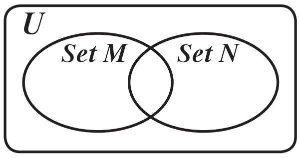


a



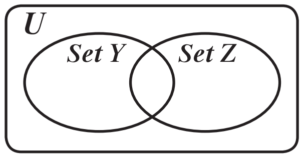
$\overline{D - E}$

b



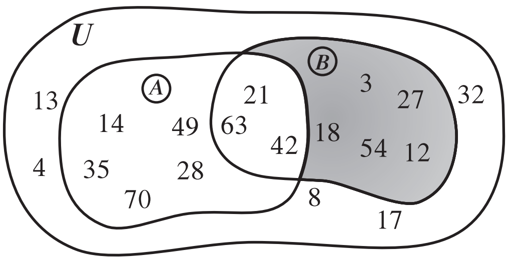
$\overline{M \cap N}$

c



Z or $\overline{Y}$ or both

6 List the members of these sets for this Venn diagram:



(i) $\overline{A \cup B} =$

(ii) $\overline{A \cap B} =$

(iii) $\overline{A - B} =$

(iv) $\overline{B - A} =$

The total number of members can be written on a Venn diagram instead of each individual element.

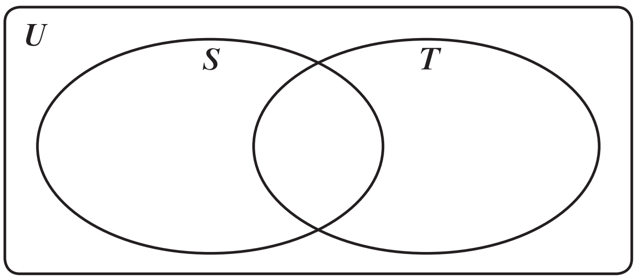
7 Twenty five year 8 students (U) were surveyed about their smartphone (S) or tablet (T) use.

Smartphone users $= n(S) = 17$

Tablet users $= n(T) = 11$

Neither $= n(\overline{S \cup T}) = 2$

Six of the students used tablets only $(T - S)$.



a Complete the Venn diagram with the number of members for each set in the appropriate place.

b Write down the number of members in each of these sets:

(i) $n(S \cup T) =$ ☐ (ii) $n(S - T) =$ ☐ (iii) $n(T - S) =$ ☐

(iv) $n(S \cup T) =$ ☐ (v) $n(S \cup T) =$ ☐ (vi) $n(\overline{S \cap T}) =$ ☐

Set diagram basics

Here are two other special types of diagrams (called Euler diagrams) that also occur

(i) • Sets with no common members are called **disjoint** or **mutually exclusive** sets.

$A = \{1, 3, 5, 7, 9\}$ $B = \{2, 4, 6, 8, 10\}$

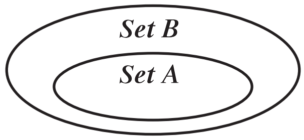
Not a Venn diagram as there is no overlap.

• $A \cap B$ set has no members.
It is a null set and we use Ø to show this. $\therefore A \cap B = \emptyset$

(ii) • One set is entirely inside another set. Called subsets (shown by $\subset$).

$A = \{3, 5\}$ $B = \{1, 2, 3, 4, 5, 6, 7\}$

The members of Set A are also in Set B.



$\therefore A \subset B$

Set A is a subset of Set B

8 **a** Indicate which of these sets are mutually exclusive or not.

(i) $Y = \{3, 6, 9, 12, 15\}$
$Z = \{8, 10, 11, 12, 18\}$
Mutually exclusive? ☐ Yes ☐ No

(ii) $W = \{$  $\}$
$X = \{$  $\}$
Mutually exclusive? ☐ Yes ☐ No

(iii) $A = \{5, 7, 11, 13, 17, 19, 23\}$
$B = \{8, 10, 12, 15, 20, 24\}$
Mutually exclusive? ☐ Yes ☐ No

(iv) $S = \{$  $\}$
$T = \{$  $\}$
Mutually exclusive? ☐ Yes ☐ No

(v) $C = \{$Even factors of $12\}$
$D = \{$Odd factors of $12\}$
Mutually exclusive? ☐ Yes ☐ No

(vi) $G = \{$A, E, I, O, U$\}$
$H = \{$C, U, F, V, D, M$\}$
Mutually exclusive? ☐ Yes ☐ No

(vii) $J = \{$Even numbers $< 10\}$
$K = \{$Even numbers $> 8\}$
Mutually exclusive? ☐ Yes ☐ No

(viii) $M = \{$Factors of $24\}$
$N = \{$Multiple of 3, $> 10\}$
Mutually exclusive? ☐ Yes ☐ No

(ix) $P = \{$Prime factor of $15\}$
$Q = \{$Prime factor of $75\}$
Mutually exclusive? ☐ Yes ☐ No

b Write down the number of members these subsets have (or label a null set if no members).

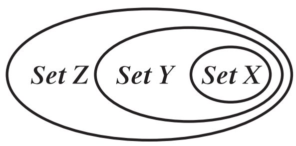
(i) $n(Y \cap Z) =$ ☐ (ii) $n(W - X) =$ ☐ (iii) $n(A \cap B) =$ ☐

(iv) $n(S \cup T) =$ ☐ (v) $n(D \cap C) =$ ☐ (vi) $n(G - H) =$ ☐

(vii) $n(J \cap K) =$ ☐ (viii) $n(M \cap N) =$ ☐ (ix) $n(P \cup Q) =$ ☐

c Which pair of sets in part **a** has one set sharing all its members with the other? ☐

9 **a** Briefly describe what this Euler diagram is saying about Sets X, Y and Z.



b How do you think this would be written using set notation? ☐

Probability and set diagrams

If there are two favourable outcomes, we use this rule:

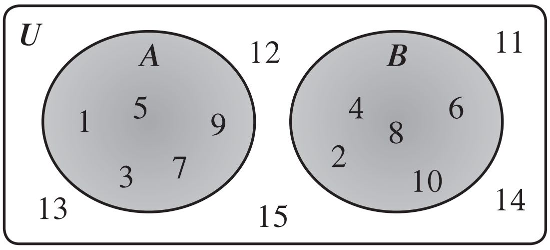
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Probability of outcome A Probability of outcome B Probability of outcome $A \cap B$

We subtract $P(A \cap B)$ to avoid counting the overlapping favourable outcomes twice.

- **Mutually exclusive events.**

  $P(A \text{ and } B) = 0$, so we simply add the probability of each favourable outcome.



$A = \{1, 3, 5, 7, 9\} \quad \therefore n(A) = 5$

$B = \{2, 4, 6, 8, 10\} \quad \therefore n(B) = 5$

$n(U) = 15$

Remember: Mutually exclusive events cannot occur at the same time

$P(A) = \dfrac{n(A)}{n(U)} = \dfrac{5}{15} = \dfrac{1}{3}$ $\quad P(A)$

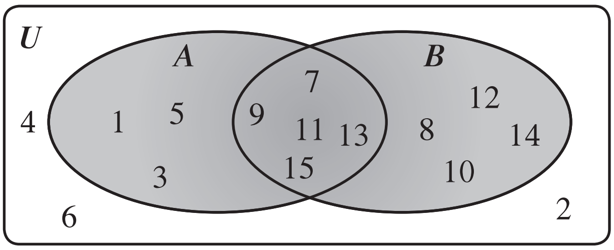
$P(B) = \dfrac{n(B)}{n(U)} = \dfrac{5}{15} = \dfrac{1}{3}$ $\quad$ Plus $P(B)$

$P(A \cap B) = \dfrac{n(A \cap B)}{n(U)} = \dfrac{0}{15} = 0$ $\quad$ Minus $P(A \cap B)$

$$\begin{aligned} \therefore P(A \text{ or } B) &= P(A) + P(B) - 0 \\ &= \frac{1}{3} + \frac{1}{3} \\ &= \frac{2}{3} \quad \text{or} \quad 66\tfrac{2}{3}\% \end{aligned}$$

- **Inclusive events.**

  $P(A \text{ and } B) \neq 0$, so we must subtract it from $P(A) + P(B)$.



$A = \{1, 3, 5, 7, 9, 11, 13, 15\} \quad \therefore n(A) = 8$

$B = \{7, 8, 9, 10, 11, 12, 13, 14, 15\} \quad \therefore n(B) = 9$

$n(U) = 15$

$P(A) = \dfrac{n(A)}{n(U)} = \dfrac{8}{15}$ $\quad P(A)$

$P(B) = \dfrac{n(B)}{n(U)} = \dfrac{9}{15} = \dfrac{3}{5}$ $\quad$ Plus $P(B)$

$P(A \cap B) = \dfrac{n(A \cap B)}{n(U)} = \dfrac{5}{15} = \dfrac{1}{3}$ $\quad$ Minus $P(A \cap B)$

$$\begin{aligned} \therefore P(A \text{ or } B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{8}{15} + \frac{3}{5} - \frac{1}{3} \\ &= \frac{4}{5} \quad \text{or} \quad 80\% \end{aligned}$$

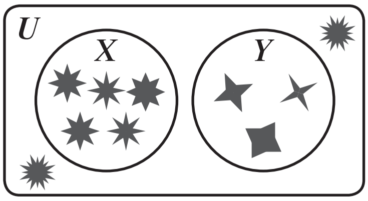
From a Venn diagram, $P(A \text{ or } B)$ can be also be calculated using $P(A \cup B) = \dfrac{n(A \cup B)}{n(U)}$. Try it!

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{n(A \cup B)}{n(U)}$$

Probability and set diagrams

1 Fill in the missing values and complete the probability calculations for these set diagrams:

a



(i) Are outcomes from sets X and Y mutually exclusive? ☐ Yes ☐ No

(ii) $n(X) = \square$ (iii) $n(Y) = \square$ (iv) $n(U) = \square$

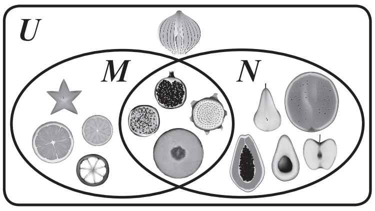
(v) $n(X \cap Y) = \square$ (vi) $n(X \cup Y) = \square$

(vii) $P(X \text{ or } Y) = P(\square) + P(\square) - P(\square) = \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

(viii) Show that you get the same result for $P(X \text{ or } Y)$ using $\frac{n(X \cup Y)}{n(U)}$ from the Venn diagram.

$$P(X \text{ or } Y) = \frac{\square}{\square}$$

b



(i) Are outcomes from sets X and Y mutually exclusive? ☐ Yes ☐ No

(ii) $n(M) = \square$ (iii) $n(N) = \square$ (iv) $n(U) = \square$

(v) $n(M \cup N) = \square$ (vi) $n(M \cap N) = \square$

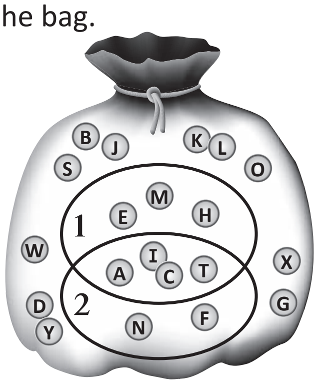
(vii) $P(X \cup Y) = P(\square) + P(\square) - P(\square) = \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

(viii) Show that you get the same result for $P(M \cup N)$ using $\frac{n(M \cup N)}{n(U)}$ from the Venn diagram.

$$P(M \cup N) = \frac{\square}{\square}$$

c (i) Are the outcomes from sets 1 and 2 in the Venn diagram below mutually exclusive? ☐ Yes ☐ No

(ii) Use two different methods to calculate $P(1 \cup 2)$ when an object is selected at random from the bag.



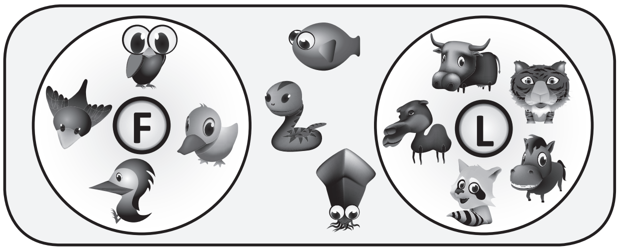
Method 1:

Method 2:

Probability and set diagrams

2 Some animals in a small sanctuary were sorted by: (F) Animals that can fly,
(L) Animals with exactly four legs.

a



(i) $n(F) =$ ☐

(ii) $n(L) =$ ☐

(iii) $n(U) =$ ☐ (The number of animals)

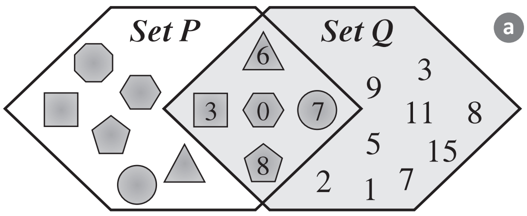
b Briefly explain why the two categories (F) and (L) do not overlap in the diagram.

c If an animal was selected at random, what is the probability it could fly as a percentage?

d There are three animals in neither category. Use this to calculate $P(\overline{F \text{ or } L})$.

Remember:
The bar over the top means the complement.

3 The Venn diagram below shows all the elements in a box containing a mixture of shapes, numbers and shapes with numbers on them only.



a Write down the number of members in each of these sets:

(i) $n(P) =$ ☐ (ii) $n(Q) =$ ☐

(iii) $n(P \text{ and } Q) =$ ☐ (iv) $n(P \text{ or } Q) =$ ☐

b Write a rule you need to use to calculate $P(P \text{ or } Q)$.

c One of the elements were selected randomly from the box. Calculate $P(P \text{ or } Q)$.

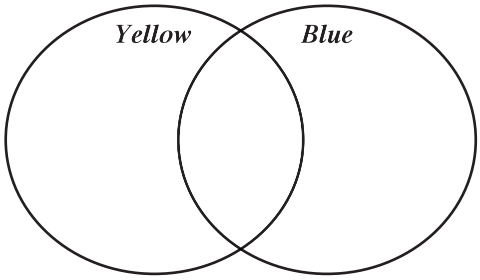
e Briefly explain why the probability of selecting a shape (Set P) or a number (Set Q) equals 1.

d What probability would the calculation $\dfrac{n(P \cap Q)}{n(P \cup Q)} \times 100\%$ be finding?

Probability and set diagrams

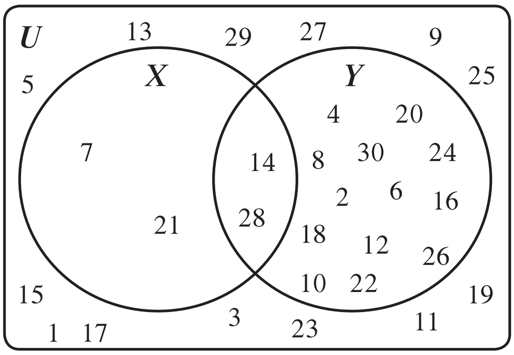
4 **a** Use the information below to fill in the Venn diagram:

- 8 of the objects are blue only.
- 10 Green objects are a combination of yellow and blue.
- 40% of all the objects contain yellow.



b What is the probability of selecting a green $(Yellow \cap Blue)$ object at random?

5 A bag contains discs, each one with a different number from 1 through to 30.
This Venn diagram shows the chosen subsets in the question and where they overlap.



$X =$ Multiples of 7 up to 28

$= \{7, 14, 21, 28\}$

$\therefore n(X) = 4$

$Y =$ Even integers up to 30

$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$

$\therefore n(Y) = 15$

If one disc is drawn at random, what is the probability it contains:

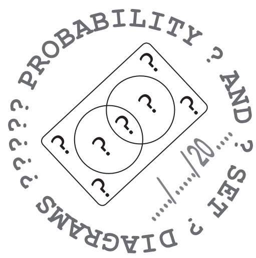
a A multiple of 7?

b An even number?

c An even number and a multiple of 7?

d Either an even number or a multiple of 7?

e An even number that is not a multiple of 7?

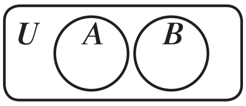


Probability and set diagrams

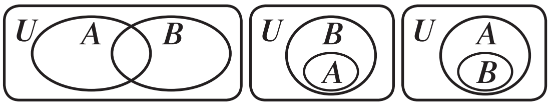
Complementary probabilities can be simple to find using Venn diagrams

The complementary probability $P(\overline{A})$ is represented here in these various diagrams:

- Mutually exclusive

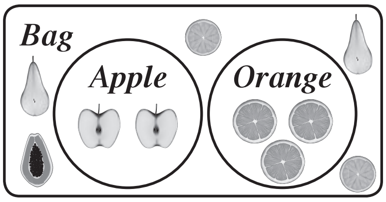


- Inclusive



$$P(\overline{A}) = \frac{n(\overline{A})}{n(U)} = 1 - P(A)$$

They show all the members of the set that are NOT members of Set A.
What is the probability of not selecting an orange from the bag of fruit shown below?

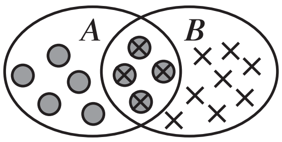


$$P(\overline{Orange}) = \frac{n(\overline{Orange})}{n(Bag)} = \frac{2+5}{10} = \frac{7}{10} \text{ or } 70\%$$

Same as: $P(\overline{Orange}) = 1 - P(Orange) = 1 - \frac{3}{10} = \frac{7}{10}$

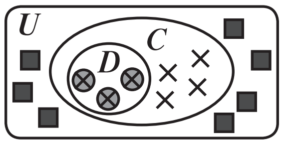
6 Calculate the given complementary probability for each of these Venn diagrams:

a



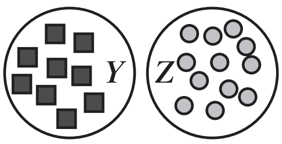
$P(\overline{A}) =$

b



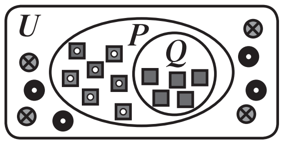
$P(\overline{C}) =$

c



$P(\overline{Y}) =$

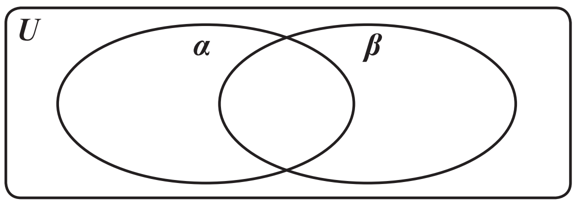
d



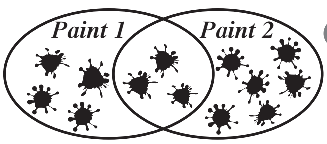
$P(\overline{Q}) =$

7 Use this information to complete the Venn diagram.

- Set $\alpha =$ Positive multiples of 4 less than 20
- Set $\beta =$ Positive multiples of 6 less than 20
- $P(\overline{\alpha \cup \beta}) = \frac{1}{20}$



8 Drops of two different paints were spilt onto a bench, some of which mixed together.

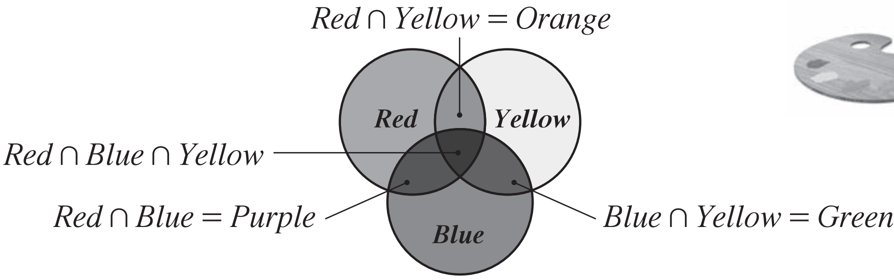


a Calculate the probability that a drop of paint has no paint 2 in it.

b Another drop of Paint 1 is spilt and it mixes with one of the drops that contained paint 2 only.
Has the probability that a drop on the bench does not contain paint 2 in it changed?
Draw a new Venn diagram to help.

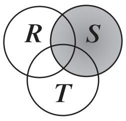
More Venn diagrams

Venn diagrams can have three or more overlapping sets.
Look at the three primary colours red, yellow and blue and how they mix.
All the same rules and symbols apply to three-set Venn diagrams as before.

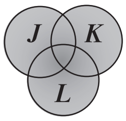


Here are some three-set Venn diagram examples and the related set notation

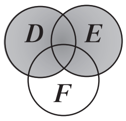
(i) Use set notation to describe the shaded region of the Venn diagrams below:



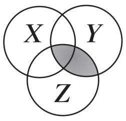
S
Members of Set S.



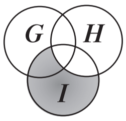
$J \cup K \cup L$
Union of all three sets
J **or** K **or** L



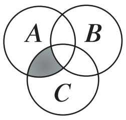
$D \cup E$
Union of Set D and Set E
D **or** E



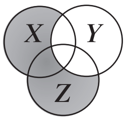
$Y \cap Z$
Intersection of Set Y and Set Z
Y **and** Z



$I - H$
Members of Set I that are not also in Set H
I **not** H

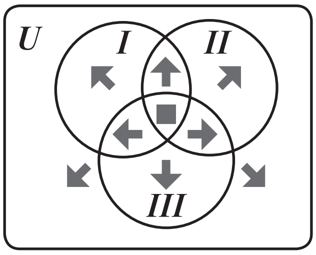


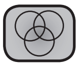
$(A \cap C) - B$
Intersection of Set A and Set C, minus the members which are also in Set B.
A **and** C, **not** B

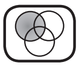


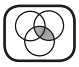
$(X \cup Z) - Y$
Union of Set X and Set Z, minus the members which are also in Set Y.
X **or** Z, **not** Y

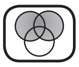
(ii) For each set list the members and their total number from this Venn diagram:

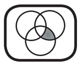


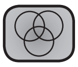
a $U =$  $= \{$↖, ↑, ↗, ←, ■, →, ↙, ↓, ↘$\}$ $\therefore n(U) = 9$

b $I =$  $= \{$↖, ↑, ←, ■$\}$ $\therefore n(I) = 4$

c $(II \cap III) =$  $= \{$■, →$\}$ $\therefore n(II \cap III) = 2$

d $(I \cup II) =$  $= \{$↖, ↑, ↗, ←, ■, →$\}$ $\therefore n(I \cup II) = 6$

e $(II \cap III) - I =$  $= \{$→$\}$ $\therefore n((II \cap III) - 1) = 1$

f $\overline{II} =$  $= \{$↖, ←, ↙, ↓, ↘$\}$ $\therefore n(\overline{II}) = 5$

(iii) Calculate the percentage probability that a member of these sets is randomly chosen.

a Probability of members in Set II

$$\frac{n(I)}{n(U)} \times 100\% = \frac{4}{9} \times 100\% = 44.\dot{4}\%$$

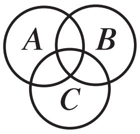
b Probability of members in Set $(\overline{I \cup II})$

$$\frac{n(\overline{I \cup II})}{n(U)} \times 100\% = \frac{3}{9} \times 100\% = 33\frac{1}{3}\%$$

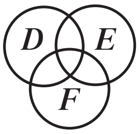
More Venn diagrams

1 Shade the areas on these Venn diagrams that match the given set notations.

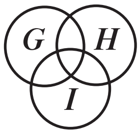
a C



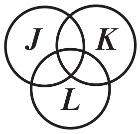
b $D \cup E$



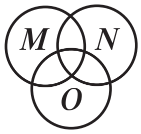
c $G \cap H$



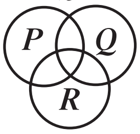
d $J - L$



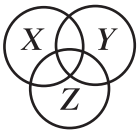
e $M \cap N \cap O$



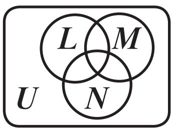
f $\overline{P \cap Q \cap R}$



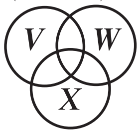
g $\overline{X}$



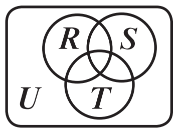
h $\overline{L}$



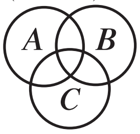
i $\overline{(W \cap X)}$



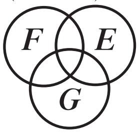
j $\overline{(R \cup S)}$



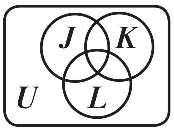
k $(A \cap B) - C$



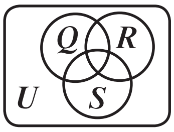
l $(E \cup G) - F$



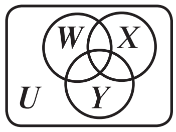
m $\overline{K - J}$



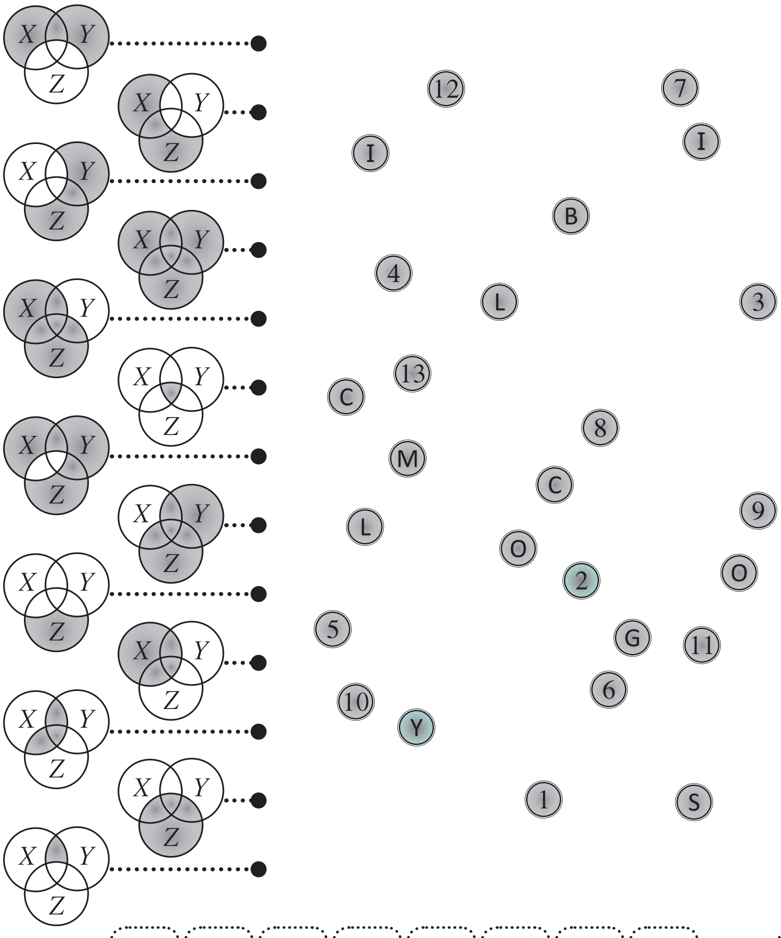
n $\overline{(Q \cup S) - R}$



o $\overline{(W \cap X) - Y}$



2 Link the descriptions below with the matching Venn diagram using straight lines to reveal the title of the book John Venn wrote to introduce Venn diagrams back in 1881.

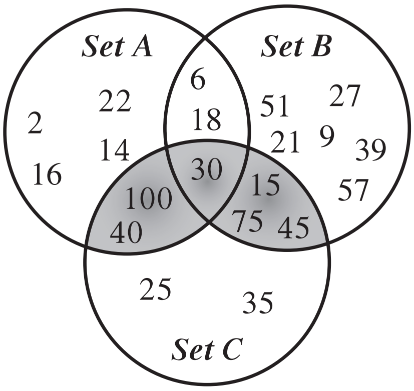


- Y or Z but not X
- Either X or Z
- X or Y but not Z
- Y or Z
- X and Y but not Z
- Only set X
- X and Y and Z
- Either (X and Z) or (X and Y)
- Not Y
- $\overline{X \text{ or } Y}$
- Everything in X, Y or Z
- Not $\overline{Z}$
- Not (X and Z)

1	2	3	4	5	6	7	8		9	10	11	12	13

More Venn diagrams

3 This Venn diagram contains three sets.



a Use set notation to list the members of each Set A, B and C.

Set $A =$

Set $B =$

Set $C =$

b How many members are there in each set?

$n(Set\ A) =$ ☐ $n(Set\ B) =$ ☐ $n(Set\ C) =$ ☐

c $n(A \cup B \cup C) =$ ☐

d (i) Does $n(A \cup B \cup C) = n(Set\ A) + n(Set\ B) + n(Set\ C)$? ☐ Yes ☐ No
(ii) Explain your answer below:

e List the members and totals for each of these:

(i) $(A \cup C) =$ $\therefore n(A \cup C) =$ ☐
(All the different members in Set A or Set C)

(ii) $(B \cap C) =$ $\therefore n(B \cap C) =$ ☐
(All the members that are in Set B and Set C)

(iii) $(A \cap B \cap C) =$ $\therefore n(A \cap B \cap C) =$ ☐
(All the members that can be found in Set A or Set B or Set C or all)

(iv) $(C - A) =$ $\therefore n(C - A) =$ ☐
(All the members in Set C that are not also members of Set A)

(v) $(\overline{A \cup B}) =$ $\therefore n(\overline{A \cup B}) =$ ☐
(All the members that are not in Set A or Set B)

f A number is randomly chosen. Calculate the percentage probability it is from one of these sets:

(i) $P(A \cup C)$
$$= \frac{n(A \cup C)}{n(A \cup B \cup C)} = \frac{14}{20} = 70\%$$

(ii) $P(B \cap C)$

(iii) $P(B)$

(iv) $P(\overline{B})$

(v) $P(\overline{A \cap C})$

(vi) $P(A \cap B \cap C)$

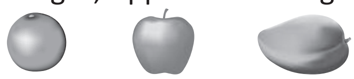
(vii) $P((A \cap C)$ or $(B \cap C))$

(viii) $P(A$ or C but not $B)$

More Venn diagrams

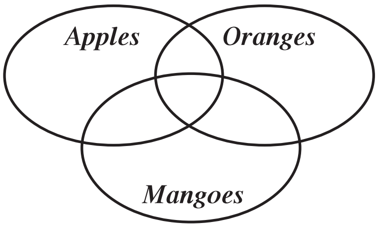
4 A class of students were surveyed about whether or not they liked these three fruits: Oranges, Apples and Mangoes.

Students who liked:



- Only one piece of fruit
  Oranges $= 8$
  Apples $= 4$
  Mangoes $= 7$
- Two pieces of fruit
  Oranges and Apples $= 8$
  Oranges and Mangoes $= 4$
  Apples and Mangoes $= 7$
- All three peices of fruit
  Oranges, Apples and Mangoes $= 8$

a Put this information into the Venn diagram.



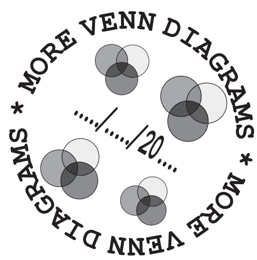
b Calculate:
(i) $n(Apples) =$
(ii) $n(Oranges) =$
(iii) $n(Mangoes) =$
$\therefore$ The most liked fruit is the

c Write the probability for selecting a surveyed student at random who liked oranges as a simplified fraction.

d Write the probability for selecting a surveyed student at random who did not like Apples to 2 decimal places.

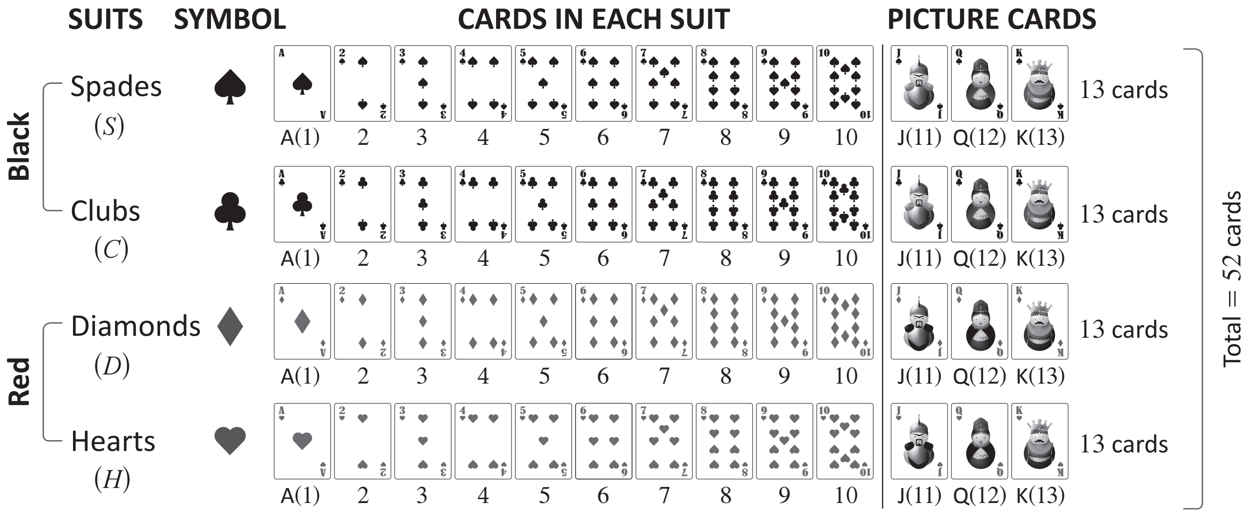
e Fourteen more students were surveyed. Eight liked Apples and Mangoes while the remaining six like all three pieces of fruit.
(i) Has the number of students who don't like Apples ☐ changed ☐ stayed the same?
(ii) Does this mean the probability for selecting a surveyed student at random who does not like apples has increased, decreased or remained the same? Explain your answer.

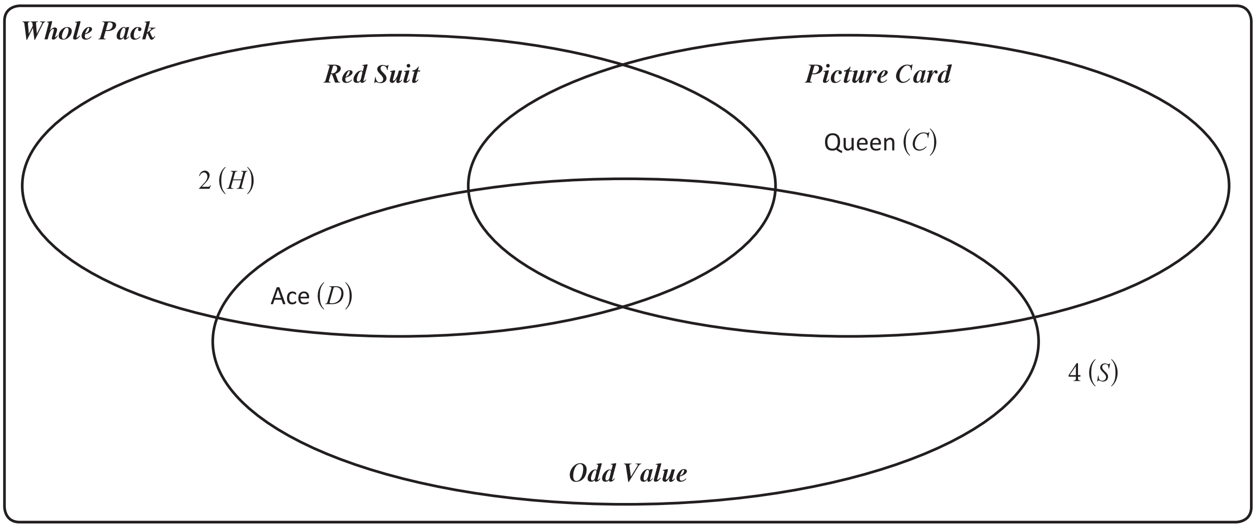
f Recalculate the probabilities of parts **c** and **d** including these fourteen newly surveyed students.



More Venn diagrams

5 **a** Use the pack of card information here to complete the partially filled in Venn diagram.





b A card is selected randomly from the pack. Use the Venn diagram to calculate the probability that the card belongs to the following sets accurate to 2 decimal places:

(i) $P(\textit{Red suit})$

(ii) $P(\textit{Picture card})$

(iii) $P(\textit{Red suit}$ or $\textit{Odd valued}) = P(\textit{Red} \cup \textit{Odd})$

(iv) $P(\textit{Red} \cap \textit{Odd} \cap \textit{Picture})$

(v) $P(\overline{\textit{Red} \cup \textit{Odd} \cup \textit{Picture}})$

(vi) $P(\overline{\textit{Picture card} \cap \textit{Odd}})$

(vii) $P(\textit{Red card} - \textit{Picture card})$

(viii) $P(\overline{\textit{Black picture card}})$

Reflection Time

Reflecting on the work covered within this booklet:

- What useful skills have you gained by learning how to calculate probabilities?

- Write about one or two ways you think you could apply probability calculations to a real life situation.

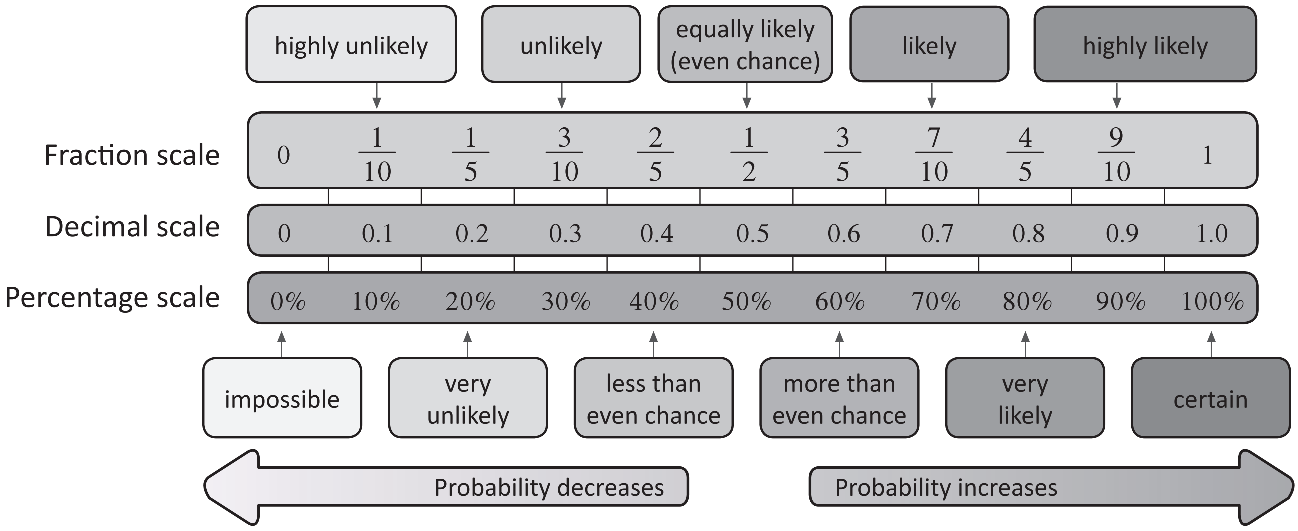
- If you discovered or learnt about any shortcuts to help with probability calculations, using two-way tables or Venn diagrams, (or simply some other cool facts), jot them down here:

Here is a summary of some important things to remember for Probability

Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} \quad \begin{array}{l}\text{Size of the favourable outcomes space}\\ \text{Size of the sample space}\end{array}$$

$$\text{Probability of the event } (E) = \frac{\text{The number } (n) \text{ of times the even } (E) \text{ can happen}}{\text{The number } (n) \text{ of outcomes in the sample space } (S)}$$



Complementary Events

The probability of an event not happening: $P(\overline{E}) = 1 - P(E)$

Independent and Dependent Events

- **Independent:** The outcomes of one event does not affect the outcomes of the other.
  Eg: Flipping two coins.
- **Dependent:** The outcome of one event affects the outcome of the other.
  Eg: Popping the balloon in a bunch that contains a prize.

Mutually Exclusive and Inclusive Events

- Mutually exclusive events cannot happen at the same time.
  When rolling a die: A number that is odd or is a multiple of two cannot happen at the same time.
- Inclusive events can happen at the same time.
  When rolling a die: A number that is odd or is a multiple of three can happen at the same time.

Relative Frequency

The fraction of observed results in any experiment/collection of trials is called the relative frequency.

Number of times it happens

$$\text{Relative frequency} = \frac{\text{The frequency of the outcome being observed}}{\text{The number of trials completed}}$$

Two-Way Table Probability Calculations

A good way to recorded results from a probability experiment containing two sets of outcomes

		Outcome 1					
		1	2	3	4	Total	
Outcome 2	A	13	8	12	14	47	← Total A's
	B	15	14	8	16	53	← Total B's
	Total	28	22	20	30	100	← Sum of the column/row totals should both equal the total number of trials.
		↑ Total 1s	↑ Total 2s	↑ Total 3s	↑ Total 4s		

The rules for simple Venn diagrams can be extended to more than just two sets.

$$\text{Relative frequency of } (Outcome\ 1, Outcome\ 2) = \frac{n(Outcome\ 1, Outcome\ 2)}{n(Trials)}$$

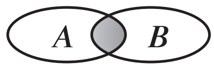
Set Diagram Basics and Probability

Euler diagrams are used to represent sets of data that do not overlap or groupings within a set.
Venn diagrams show the members of collected data sets and where they overlap.

Intersection $(\cap)$

- Where Sets A and B have the same members.

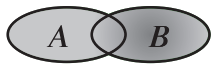
$$P(A \cap B) = P(A \text{ and } B) = \frac{n(A \cap B)}{n(Members\ in\ the\ diagram)}$$



Union $(\cup)$

- All the members that are in either Set A **or** Set B **or** both.

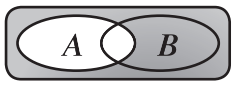
$$P(A \cup B) = P(A \text{ or } B) = \frac{n(A \cup B)}{n(Members\ in\ the\ diagram)}$$



Complement $(\overline{A})$

- Everthing that is **not** in the Set A.

$$P(\overline{A}) = \frac{n(\overline{A})}{n(Members\ in\ the\ diagram)} = 1 - P(A)$$

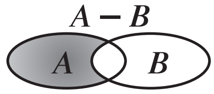


Difference $(-)$

- The difference between two sets (relative complement). Shows what is unique to a set. The order is important

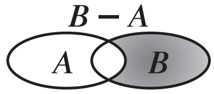
$$P(A - B) = P(A \text{ not } B) = \frac{n(A - B)}{n(Members\ in\ the\ diagram)}$$

$A - B$



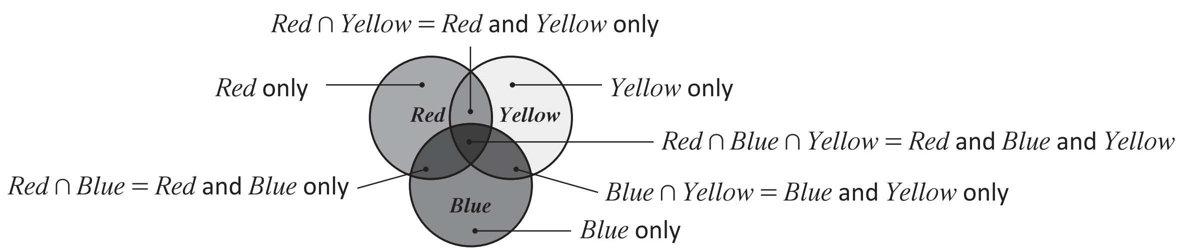
$$P(B - A) = P(B \text{ not } A) = \frac{n(B - A)}{n(Members\ in\ the\ diagram)}$$

$B - A$



More Venn Diagrams

The rules for simple Venn diagrams can be extended to more than just two sets.



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