Probability

Solutions

Curriculum Ready

Mathletics

www.mathletics.com

Page 3 questions

Theoretical probability

1 **a**

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$n(S) = n(Numbers) = 8$

b

$S = \{R, O, P, Y, G\}$

$n(S) = n(Colours) = 5$

c

$S = \{No, Yes, Maybe, Ask\ again\}$

$n(S) = n(Options) = 4$

2 **a** $n(E) = 2, \quad n(S) = 25$

$P(E) = \frac{2}{25} = 0.08$

b $n(White\ flowers) = 10, \quad n(Flowers) = 14$

$P(White\ flowers) = \frac{10}{14} = 0.71$ (to 2 d.p.)

c $n(Brown) = 7, \quad n(Cards) = 16$

$P(Brown\ card) = \frac{7}{16} = 0.44$ (to 2 d.p.)

d $n(Odd\ numbers) = 12, \quad n(Numbers) = 33$

$P(Odd\ numbers) = \frac{12}{33} = 0.36$ (to 2 d.p.)

3 **a** $n(E) = 1, \quad n(S) = 2$

$P(E) = \frac{1}{2} \times 100\% = 50\%$

b $n(E) = 3, \quad n(S) = 25$

$P(E) = \frac{3}{25} \times 100\% = 12\%$

c $n(E) = 36, \quad n(S) = 48$

$P(E) = \frac{36}{48} \times 100\% = 75\%$

d $n(E) = 5, \quad n(S) = 8$

$P(E) = \frac{5}{8} \times 100\% = 62\frac{1}{2}\%$

4 **a** $P(E) = \frac{1}{4}, \quad n(S) = 12$

$\therefore n(E) = \frac{1}{4} = \frac{n(E)}{12} \quad \therefore n(E) = 3$

b $P(E) = 30\% \left(\text{i.e. } \frac{3}{10}\right), \quad n(E) = 15$

$\therefore n(S) = \frac{3}{10} = \frac{15}{n(S)} \quad \therefore n(S) = 50$

c $P(Orange) = 0.6 \left(\text{i.e. } \frac{6}{10}\right), \quad n(Oranges) = 21$

$\therefore P(Orange) = \frac{3}{5} = \frac{21}{n(Fruit)}$

$\therefore n(Fruit) = 35$

d $n(Animals) = 12, \quad P(Duck) = 75\%$

$\therefore P(Duck) = \frac{3}{4} = \frac{n(Ducks)}{12}$

$\therefore n(Ducks) = 9$

Page 4 questions

Theoretical probability

5 **a** (i) $n(9) =$ 3 (ii) $n(B) =$ 2 (iii) $n(Letter) =$ 9 (iv) $n(Number) =$ 11

(v) $n(H \text{ or } 6) =$ 7 (vi) $n(Number < 9) =$ 8 (vii) $n(7) =$ 0 (viii) $n(Even) =$ 6

b (i) $S = \{5, 5, 9, 9, 9, 2, 2, 6, 6, 6, 6, K, H, H, H, B, B, T, T, T\}$

(ii) $n(S) =$ 20

c (i) $P(9) = \frac{n(9)}{n(S)} = \frac{3}{20}$ (ii) $P(Even) = \frac{n(Even)}{n(S)} = \frac{3}{10}$ (iii) $P(B) = \frac{1}{10}$

(iv) $P(Number < 6) = \frac{1}{5}$ (v) $P(H \text{ or } B) = \frac{1}{4}$ (vi) $P(6 \text{ or } 9 \text{ or } T) = \frac{1}{2}$

6 **a** $S = \{No\ ball, No\ ball, Ball\}$ $n(S) = 3$

b $P(No\ ball) = \frac{2}{3} \times 100\% = 66\frac{2}{3}\%$ $P(Ball) = \frac{1}{3} \times 100\% = 33\frac{1}{3}\%$

c $P(No\ ball) + P(Ball) = 66\frac{2}{3}\% + 33\frac{1}{3}\% = 100\%$

Page 5 questions

Theoretical probability

7 **a** (i) $n(Red\ numbers) =$ 6 (ii) $n(Multiples\ of\ 3) =$ 4 (iii) $n(Number < 9) =$ 9

(iv) $n(Even\ numbers) =$ 7 (v) $n(Single\ digit\ numbers) =$ 10

b (i) $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ (ii) $n(S) =$ 13

Page 5 questions

Theoretical probability

7 c (i) $P(\textit{Black number}) = \frac{6}{13} = 0.462$ to 3 d.p.

(ii) $P(8) = \frac{1}{13} = 0.077$ to 3 d.p.

(iii) $P(\textit{Prime number}) = \frac{5}{13} = 0.385$ to 3 d.p.

(iv) $P(\textit{Multiple of } 5) = \frac{2}{13} = 0.154$ to 3 d.p.

Page 6 questions

Theoretical probability

8 a $n(S) = 16$

b (i) $n(\textit{Even sum}) = 8$

(ii) $n(\textit{Sum of } 5) = 4$

(iii) $n(\textit{Sum of } 6) = 3$

(iv) $n(\textit{Sum of } 3) = 2$

(v) $n(\textit{Sum} < 5) = 6$

(vi) $n(\textit{At least one } 3) = 7$

(vii) $n(\textit{Sum} > 2) = 15$

(viii) $n(\textit{Sum is a prime}) = 9$

Page 6 questions

Theoretical probability

8 c (i) $P(Even\ sum) =$ $0.5, 50\%, \frac{1}{2}$

∴ Even chance

(ii) $P(Sum\ of\ 5) =$ $0.25, 25\%, \frac{1}{4}$

∴ Unlikely/ very unlikely

(iii) $P(Sum\ of\ 6) =$ $0.1875, 18.75\%, \frac{3}{16}$

∴ Very unlikely

(iv) $P(Sum\ of\ 3) =$ $0.125, 12.5\%, \frac{1}{8}$

∴ Highly unlikely

(v) $P(Sum < 5) =$ $0.375, 37.5\%, \frac{3}{8}$

∴ Less than even chance/ unlikely

(vi) $P(At\ lease\ one\ 3) =$ $0.4375, 43.75\%, \frac{7}{16}$

∴ Less than even chance

(vii) $P(Sum > 2) =$ $0.9375, 93.75\%, \frac{15}{16}$

∴ Highly likely

(viii) $P(Sum\ is\ a\ prime) =$ $0.5625, 56.25\%, \frac{9}{16}$

∴ More than even chance

Page 8 questions

Complementary events

1 a $\overline{Event} =$ Turning light switch off ***or*** not turning it on

b $\overline{Event} =$ standing up ***or*** not sitting down

c $\overline{Event} =$ Answering correctly ***or*** not answering incorrectly

d $\overline{Event} =$ closing a jar ***or*** not opening a jar

e $\overline{Event} =$ not travelling by train

f $\overline{Event} =$ Rolling an odd number ***or*** not rolling an even number

g $\overline{Event} =$ Hearing a sound

h $\overline{Event} =$ Understanding the task

2 a $P(Broken)$ or $P(\overline{Fixed})$

b $P(\overline{Here})$ or $P(Away)$ or $P(There)$

c $P(Healthy)$ or $P(\overline{Sick})$

d $P(Polite)$ or $P(\overline{Impolite})$

e $P(Big)$ or $P(\overline{Small})$

f $P(\overline{Did\ see\ it})$ or $P(You\ didn't\ see\ it)$

Page 8 questions

Complementary events

3 **a** $P(A\ Blue\ Card) = \frac{1}{4}$

$\therefore P(\overline{A\ blue\ card}) = 1 - \frac{1}{4}$

$= \frac{3}{4}$

b $P(Winning) = 65\%$

$\therefore P(\overline{Losing\ \textbf{or}\ (Winning)}) = 100\% - 65\%$

$= 35\%$

Page 9 questions

Complementary events

4 **a** $P(Not\ blue) = 1 - \frac{1}{3} = \frac{2}{3}$

b $P(Poor\ reception) = 1 - 85\% = 15\%$

c $P(\overline{Arriving\ on\ time}) = 1 - 0.30 = 0.70$

d $P(\overline{Raining\ tomorrow}) = 1 - \frac{2}{5} = \frac{3}{5}$

e $P(Parrot\ talking) = 1 - 0.17 = 0.83$

f $P(Have\ green\ eyes) = 1 - 74.4\% = 25.6\%$

5 **a** $P(\$25\ mobile\ phone\ credit)$

$= \frac{4}{50} = \frac{2}{25}$

b $P(\overline{\$25\ mobile\ phone\ credit})$

$= 1 - \frac{2}{25} = \frac{23}{25}$

c $P(New\ prepaid\ mobile\ phone)$

$= \frac{1}{50}$

d $P(\overline{New\ prepaid\ mobile\ phone})$

$= 1 - \frac{1}{50} = \frac{49}{50}$

e $P(\$5\ or\ \$10\ mobile\ phone\ credit)$

$= \frac{30 + 15}{50} = \frac{45}{50} = \frac{9}{10}$

f $P(\overline{\$25\ credit\ or\ a\ new\ prepaid\ mobile\ phone})$

$= 1 - \frac{4 + 1}{50} = 1 - \frac{5}{50} = \frac{9}{10}$

g They have the same probability value, and since **f** is a complementary probability, then the two events (picking $\$5$ or $\$10$ mobile phone credit) and (picking $\$25$ credit or a new prepaid mobile phone) are complementary events.

Page 10 questions

Complementary events

6 **a** $P(\text{Shaded puzzle piece}) = \frac{3}{25} \times 100\% = 12\%$

b $P(\overline{\text{Corner puzzle piece}}) = \left(1 - \frac{4}{25}\right) \times 100\% = 84\%$

c $P(\text{Edge puzzle piece}) = \frac{17}{25} \times 100\% = 68\%$

d $P(\overline{\text{Edge puzzle piece}}) = 100\% - 68\% = 32\%$

e $P(\overline{\text{Puzzle piece that touches a shaded piece once solved}}) = \left(1 - \frac{10}{25}\right) \times 100\% = 60\%$

7 **a** 80 seconds ÷ 2 seconds per pass = 40 passes of the parcel

$\therefore 40 \div 8 = 5$

∴ You will have held it five times.

b $P(\overline{\text{Holding the parcel}}) = 1 - P(\text{Holding the parcel})$

$$= 1 - \frac{5}{40}$$

$$= \frac{7}{8}$$

c $P(\text{Winning the main prize}) = 1 - P(\overline{\text{Holding the parcel}})$

$$= 1 - \frac{7}{8}$$

$$= \frac{1}{8}$$

$$= 0.125$$

Page 12 questions

Independent and dependent events

1. Identify each of these as dependent or independent events by ticking the right term.

		Independent	Dependent
a	Flipping two coins	✓	
b	Flicking a number spinner and selecting a numbered card at random from a pack.	✓	
c	Selecting two blunt pencils from a pencil case at the same time.		✓
d	Picking two out of three cups (one after the other) to see which one contains a hidden ball.		✓
e	Selecting two green marbles if the first marble was returned to the bag before selecting the second one.	✓	
f	Guessing correctly the first two numbers to be drawn in a game of bingo.		✓
g	Randomly selecting seven tiles in a word game, then replacing and selecting another seven tiles.	✓	
h	Two different people opening their books to the exact same page as each other.	✓	
i	Two sheep giving birth to lambs on the same day.	✓	
j	Guessing who will finish in the first two places of a race.		✓

2. a Randomly selecting an even number from one bag and a green cube from the other.

   b Randomly selecting cubes with the same colour after replacing the first cube before selecting the second one.

   c Randomly selecting two cubes and them both being yellow coloured.

   d Randomly selecting two odd numbered cards from the bag without replacing the first card drawn.

Page 13 questions

Independent and dependent events

3 a (i) ☑ Independent ☐ Dependent

(ii) Roll the die twice to record the sum, only if an odd number occurs on the first roll.
The second role (and sum) is now dependent on the outcome of the first roll.

b (i) ☐ Independent ☑ Dependent

(ii) Replacing the first before selecting the second one.
The result of the second selection will now be independent of the first one.

c (i) ☐ Independent ☑ Dependent

(ii) Vaneeta changes the number she is thinking of before each guess.
There are always 20 possible answers for each guess, so each guess is now independent of the previous one.

d (i) ☑ Independent ☐ Dependent

(ii) Rolling a four sided die when the spinner stops on a certain colour.
The roll of the die is now dependent on the outcome of the spinner.

e (i) ☐ Independent ☑ Dependent

(ii) Selecting any three numbers from a bag with replacement.
There is no change to the number of favourable outcomes or sample space, so each selection is now independent.

f (i) ☑ Independent ☐ Dependent

(ii) Selecting two keys from the same set of keys that will open a lock without replacement.
The number of keys to choose from on the second selection will have reduced.

Page 15 questions

Mutually exclusive and inclusive events

1 a Flipping a Head or Tail on two different coins.

☐ Mutually exclusive ☑ Inclusive

b A light switch in the 'on' or 'off' position.

☑ Mutually exclusive ☐ Inclusive

c Winning first or second prize in a local raffle with one ticket.

☑ Mutually exclusive ☐ Inclusive

d Winning first or second prize in a local raffle with two tickets.

☐ Mutually exclusive ☑ Inclusive

2 a Here are two possible answers
- Randomly selecting a card that contains a prime number or a multiple of 4.
- Randomly selecting a card that is a multiple of 3 or a multiple of 5

b Here are two possible answers
- Randomly selecting a card that contains a prime number or an odd number.
- Randomly selecting a card with a number greater than 3 but less than 9.

3 a Here are some possible answers
- Randomly selecting 2 orange or 2 blue marbles from Box A
  (You can only get 2 Orange or 2 blue marbles if selection is with replacement. So getting two of these colours from box A cannot happen at the same time).
- Randomly selecting 2 green or 2 yellow marbles from Box B.
  (You can only get 2 green or 2 yellow marbles if selection is with replacement. So getting two of these colours from box B cannot happen at the same time).

b Here are some possible answers
- Randomly selecting a yellow marble, one from each box.
- Randomly selecting two black marbles (with or without replacement) from Box B
- Randomly selecting two yellow marbles (with or without replacement)from Box A.

Page 16 questions

Mutually exclusive and inclusive events

4 a A student selected from the class has either brown hair or brown eyes.

☐ Exclusive Or ☑ Inclusive Or ☐ Inclusive And

b Dropping a cup and spilling all the contents.

☐ Exclusive Or ☐ Inclusive Or ☑ Inclusive And

c One of two teachers selected randomly in a school catches public transport to school.

☐ Exclusive Or ☑ Inclusive Or ☐ Inclusive And

d Boiling and freezing a container of water.

☑ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

e A person selected at random is either sitting down or standing up.

☑ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

f Rolling a number larger than 5 and an even number on a normal 6-sided die.

☐ Exclusive Or ☐ Inclusive Or ☑ Inclusive And

g Spelling a word correctly and using it properly in a sentence.

☐ Exclusive Or ☑ Inclusive Or ☐ Inclusive And

h Selecting a red card and the number 7 from a normal pack of playing cards

☐ Exclusive Or ☑ Inclusive Or ☐ Inclusive And

i A student selected randomly during period 3 was doing Physical Education or Music.

☑ Exclusive Or ☐ Inclusive Or ☐ Inclusive And

5 A dependent event is as one that relies on the outcomes of events before it.
Inclusive events can happen at the same time.

Because a single event has no previous event to affect the outcome, there is no previous outcome for the result to be dependent upon.
A single event can be inclusive if there is more than one way that event can happen.
So best to agree with the Professor because a single event cannot be a dependent one.

Page 18 questions

Two way tables

1 a

		Ice Cream	
		With	Without
Apple Pie	Hot	Hot, With ice cream	Hot, Without ice cream
	Cold	Cold, With ice cream	Cold, Without ice cream

b

		Direction	
		Clockwise	Counter Clockwise
Spinner	Yes	Yes, Clockwise	Yes, Counter clockwise
	No	No, Clockwise	No, Counter clockwise

c

		Shorts	
		Black	White
Shirt	Yellow	Yellow shirt, Black shorts	Yellow shirt, White shorts
	Red	Red shirt, Black shorts	Red shirt, White shorts
	Orange	Orange shirt, Black shorts	Orange shirt, White shorts

2 a How many people were surveyed? 54

b How many people surveyed play both instruments? 5

c How many people surveyed play the flute? 17

d How many people surveyed can play one instrument only? 26

e $n(Play\ guitar, Do\ not\ play\ flute) =$ 14

f Change the value 23 to 28 for two 'no' responses

Page 19 questions

Two way tables

3 a $200 - (18 + 21 + 37 + 26 + 15 + 42 + 33) = 8$

b $(18 + 37 + 15 + 42) = 112$

c

	Girls	Boys
Neeuk Creek	22	23
Nooroon Plains	33	24

4

	Nothing Added	Ingredient X	Ingredient Y
Brand: A	24	24	7
Brand: B	10	15	40

Page 21 questions

Two way table probabilities

1 a

		Switch 2		
		On	Off	**Total**
Switch 1	On	4	5	9
	Off	3	8	11
	Total	7	13	20

b

		Coin 2		
		Head (H)	Tail (T)	**Total**
Coin 1	Head (H)	8	14	22
	Tail (T)	17	11	28
	Total	25	25	50

Page 21 questions

Two way table probabilities

2 **a** (i)

	White	Grey	**Total**
1	18	15	33
2	19	11	30
3	13	9	22
Total	50	35	85

(ii)

		Player 2			
		Scissors (S)	Paper (P)	Rock (R)	**Total**
Player 1	Scissors (S)	8	9	5	22
	Paper (P)	11	5	7	23
	Rock (R)	4	6	5	15
	Total	23	20	17	60

b (i) How many spins were observed on the shades and numbers spinner? 85

(ii) How many games of Scissors, Paper, Rock were recorded? 60

(iii) How many times did the spinner stop on a grey sector? 35

(iv) How many times did player 1 say 'paper'? 23

(v) How many times did a game have scissors and paper (by either player) as the result? 20

(vi) If rock beats scissors, which player won the most games when this outcome occurred? $P2$

(vii) How many times did Player 1 and Player 2 make the same symbol? 18

Page 22 questions

Two way table probabilities

3 **a** How many students were surveyed in this school? 50

b How many students surveyed selected 'C' for Question 2? 11

c What was the most common outcome for the two questions asked in this survey? No, B

d What outcome did not occur for the two questions asked in this survey? Yes, D

e What is the frequency for the outcome 'Yes, A'? 4

f What is the relative frequency for the outcome 'Yes, A'? $\frac{2}{25}$

g What is the relative frequency for an answer of 'No' to Q1 as a percentage? 76%

Page 22 questions

Two way table probabilities

4 a

		Colour		
		Red	Green	**Total**
Number	≤ 10	7	8	15
	>10	5	4	9
	Total	12	12	24

b Probability$(G, \leq 10) = \frac{1}{4} = 25\%$

Relative frequency of $(G, \leq 10) = \frac{8}{24} = 33\frac{1}{3}\%$

c The relative frequency should get closer to matching the theoretical probability value.

Page 23 questions

Two way table probabilities

5 a If you were not successful with the ring toss, you did not get a chance to pick from the lucky dip, so no lucky dip prizes.

b Relative frequency of mp3 player $= \frac{2}{50} \times 100\% = 4\%$

Relative frequency of gum $= \frac{48}{50} \times 100\% = 96\%$

c The mp3 player prize is more expensive than the bubble gum, so there are a lot less mp3 player prizes in the lucky dip.

d Relative frequency of mp3 player $= \frac{2}{80} = \frac{1}{40}$

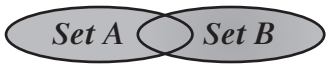
Relative frequency of gum $= \frac{78}{80} = \frac{39}{40}$

6

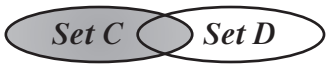
		Direction		
		Up	Down	**Total**
Number	Even	32	19	51
	Odd	16	48	64
	Total	48	67	115

Page 25 questions

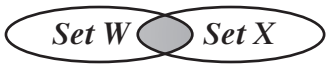
Set diagrams basics

1 **a** (i) 

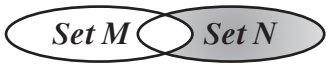
(ii) $A \cup B$

b (i) 

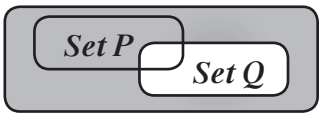
(ii) C

c (i) 

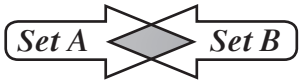
(ii) $W \cap X$

d (i) 

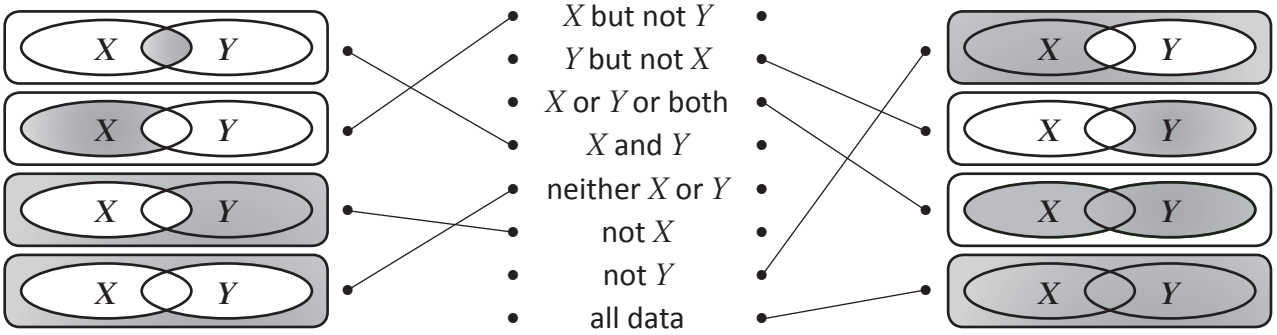
(ii) $N - M$

e (i) 

(ii) $\overline{Q}$

f (i) 

(ii) $A \cap B$

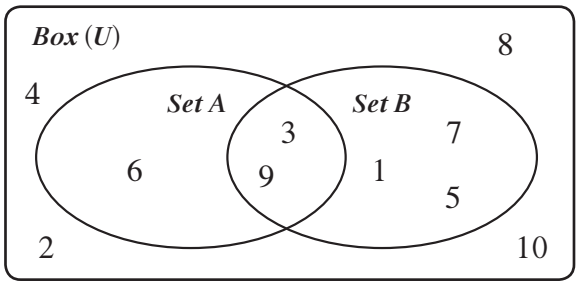
2 

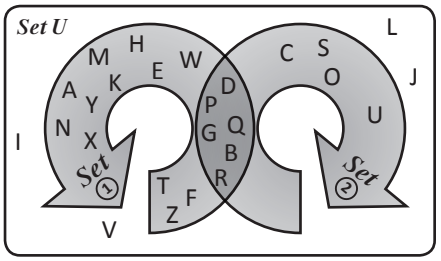
3 **a** (i) $A = \{3, 6, 9\}$

(ii) $B = \{1, 3, 5, 7, 9\}$

(iii) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(iv) $A \cap B = \{3, 9\}$

b 

4 

(i) ① $= \{A, B, D, E, F, G, H, K, M, N, P, Q, R, T, W, X, Y, Z\}$

(ii) ① $\cap$ ② $= \{B, D, G, P, Q, R\}$

(iii) $\overline{②} = \{A, E, F, H, I, J, K, L, M, N, T, V, W, X, Y, Z\}$

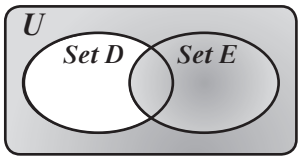
(iv) ② $-$ ① $= \{C, O, S, U\}$

(v) ① $-$ ② $= \{A, E, F, H, K, M, N, T, W, X, Y, Z\}$

Page 26 questions

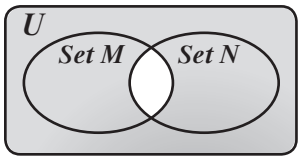
Set diagram basics

5 **a**



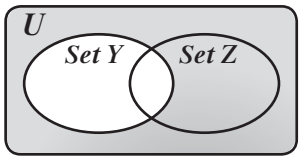
$\overline{D - E}$

b



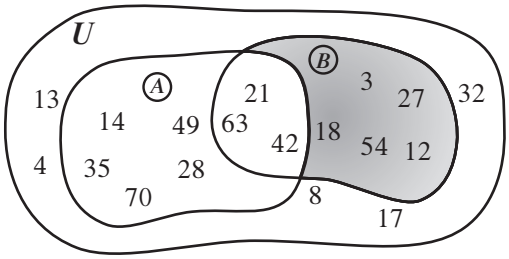
$\overline{M \cap N}$

c



Z or $\overline{Y}$ or both

6



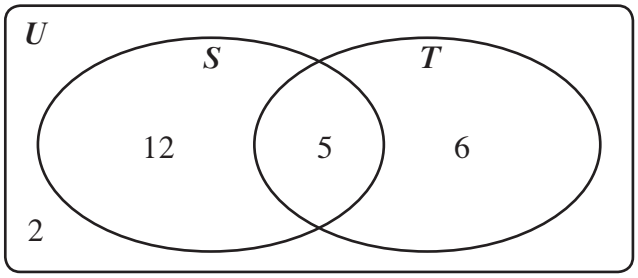
(i) $\overline{A \cup B} = \{4, 8, 13, 17, 32\}$

(ii) $\overline{A \cap B} = \{3, 4, 8, 12, 13, 14, 17, 18, 27, 28, 32, 35, 54, 70\}$

(iii) $\overline{A - B} = \{3, 4, 8, 12, 13, 17, 18, 21, 27, 32, 42, 54, 63\}$

(iv) $\overline{B - A} = \{4, 8, 13, 14, 17, 21, 28, 32, 35, 42, 49, 63, 70\}$

7 **a**



b (i) $n(S \cup T) =$ 25 (ii) $n(S - T) =$ 12 (iii) $n(T - S) =$ 19

(iv) $n(S \cup T) =$ 2 (v) $n(\overline{S \cup T}) =$ 2 (vi) $n(\overline{S \cap T}) =$ 20

Page 27 questions

Set diagram basics

8 **a** (i) $Y = \{3, 6, 9, (12), 15\}$ $Z = \{8, 10, 11, (12), 18\}$
Mutually exclusive? ☐ Yes ☒ No

(ii) $W = \{\blacksquare, \blacklozenge, \blacktriangleright, \hexagon, \bullet\}$ $X = \{\blacksquare, \pentagon, \bullet, \blacktriangle, \blacklozenge\}$
Mutually exclusive? ☐ Yes ☒ No

(iii) $A = \{5, 7, 11, 13, 17, 19, 23\}$ $B = \{8, 10, 12, 15, 20, 24\}$
Mutually exclusive? ☒ Yes ☐ No

(iv) $S = \{$  $\}$ $T = \{$  $\}$
Mutually exclusive? ☐ Yes ☒ No

(v) $C = \{\text{Even factors of } 12\}$ $D = \{\text{Odd factors of } 12\}$
Mutually exclusive? ☒ Yes ☐ No

(vi) $G = \{A, E, I, O, (U)\}$ $H = \{C, (U), F, V, D, M\}$
Mutually exclusive? ☐ Yes ☒ No

(vii) $J = \{\text{Even numbers} < 10\}$ $K = \{\text{Even numbers} > 8\}$
Mutually exclusive? ☒ Yes ☐ No

(viii) $M = \{\text{Factors of } 24\}$ $N = \{\text{Multiple of } 3, > 10\}$
Mutually exclusive? ☐ Yes ☒ No

(ix) $P = \{\text{Prime factor of } 15\}$ $Q = \{\text{Prime factor of } 75\}$
Mutually exclusive? ☐ Yes ☒ No

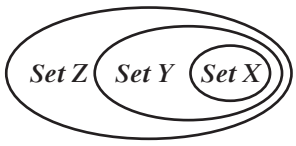
b (i) $n(Y \cap Z) =$ 1 (ii) $n(W - X) =$ 3 (iii) $n(A \cap B) =$ ∅

(iv) $n(S \cup T) =$ 6 (v) $n(D \cap C) =$ ∅ (vi) $n(G - H) =$ 4

(vii) $n(J \cap K) =$ ∅ (viii) $n(M \cap N) =$ 3 (ix) $n(P \cup Q) =$ 2

c Which pair of sets in part **a** has one set sharing all its members with the other? i.e. which pair has one set as a subset of the other?
P and Q
$P \subset Q$

9 **a**



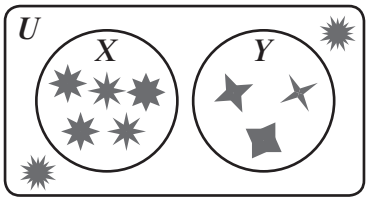
Set X is a subset of set Y which is a subset of set Z
Every member of set X is shared with set Y, and every member of set Y is shared with Set Z.

b How do you think this would be written using set notation?
$X \subset Y \subset Z$

Page 29 questions

Probability and set diagrams

1 **a**



(i) Are outcomes from sets X and Y mutually exclusive? [X] Yes [ ] No

(ii) $n(X) =$ 5 (iii) $n(Y) =$ 3 (iv) $n(U) =$ 10

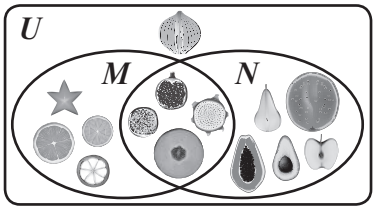
(v) $n(X \cap Y) =$ 0 (vi) $n(X \cup Y) =$ 8

(vii) $P(X \text{ or } Y) = P($ X $) + P($ Y $) - P($ X and Y $) = \frac{5}{10} + \frac{3}{10} - \frac{0}{10} = \frac{4}{5}$

(viii) Show that you get the same result for $P(X \text{ or } Y)$ using $\frac{n(X \cup Y)}{n(U)}$ from the Venn diagram.

$$P(X \text{ or } Y) = \frac{8}{10}$$

b



(i) Are outcomes from sets X and Y mutually exclusive? [ ] Yes [X] No

(ii) $n(M) =$ 8 (iii) $n(N) =$ 9 (iv) $n(U) =$ 14

(v) $n(M \cup N) =$ 13 (vi) $n(M \cap N) =$ 4

(vii) $P(X \cup Y) = P($ M $) + P($ N $) - P($ $M \cap N$ $) = \frac{8}{14} + \frac{9}{14} - \frac{4}{14} = \frac{13}{14}$

(viii) Show that you get the same result for $P(M \cup N)$ using $\frac{n(M \cup N)}{n(U)}$ from the Venn diagram.

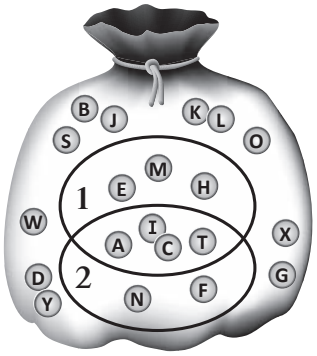
$$P(M \cup N) = \frac{13}{14}$$

Page 29 questions

Probability and set diagrams

1 **c** (i) Are the outcomes from sets 1 and 2 in the Venn diagram below mutually exclusive? ☐ Yes ☒ No

(ii) Use two different methods to calculate $P(1 \cup 2)$ when an object is selected at random from the bag.



Method 1: $n(1 \cup 2), \quad n(Bag) = 20$

$$P(1 \cup 2) = \frac{n(1 \cup 2)}{n(Bag)} = \frac{9}{20} = 45\%$$

Method 2: $n(1) = 7, \quad n(2) = 6, \quad n(1 \cap 2) = 4$

$$P(1 \cup 2) = P(1) + P(2) - P(1 \cap 2)$$

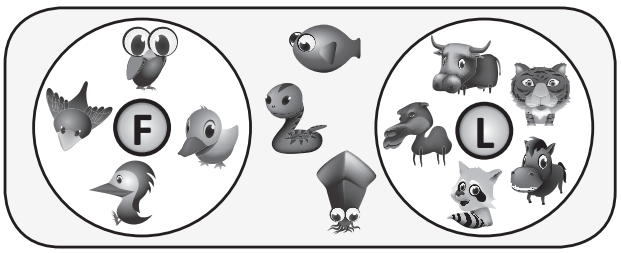
$$= \frac{7}{20} + \frac{6}{20} - \frac{4}{20}$$

$$= \frac{9}{10} = 45\%$$

Page 30 questions

Probability and set diagrams

2 **a**



(i) $n(F) =$ 4

(ii) $n(L) =$ 5

(iii) $n(U) =$ 12 (The number of animals)

b There are no four-legged flying animals/ There are no four-legged animals that can fly in the sanctuary.

c $P(F) = \frac{n(F)}{n(U)} \times 100\%$

$= \frac{4}{12} \times 100\%$

$= 33\frac{1}{3}\%$

d $P(\overline{F \text{ or } L}) = \frac{n(\overline{F \text{ or } L})}{n(U)}$

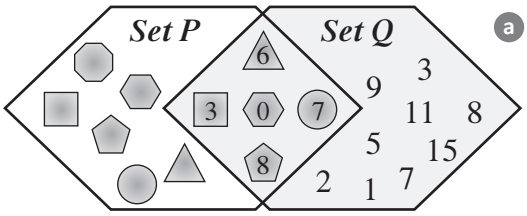
$= \frac{3}{12}$

$= \frac{1}{4}$ or 25%

Page 30 questions

Probability and set diagrams

3



a (i) $n(P) =$ 11 (ii) $n(Q) =$ 14

(iii) $n(P \text{ and } Q) =$ 5 (iv) $n(P \text{ or } Q) =$ 20

b $\therefore P(P \text{ or } Q) = P(P) + P(Q) - P(P \text{ and } Q)$ or $P(P \text{ or } Q) = \dfrac{n(P \cup Q)}{n(U)}$

c $\therefore P(P \text{ or } Q) = P(P) + P(Q) - P(P \text{ and } Q)$

$$= \frac{11}{20} + \frac{14}{20} - \frac{5}{20}$$

$$= 1 = 100\%$$

e All the elements in the box are in either set P or set Q, so the probability of selecting something from either of these sets is a certainty.

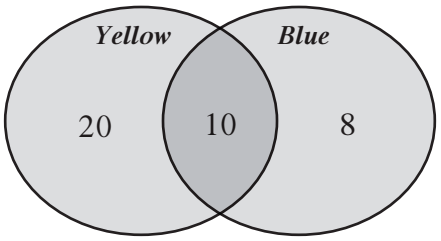
d The probability of selecting a shape that has a number written on it.

Page 31 questions

Probability and set diagrams

4

a $(100\% - 40\%) = 60\%$ objects contain Blue
$\therefore 60\% = 18$ objects
$\therefore 40\% = (18 \div 60) \times 100 = 30$ objects containing yellow.
$\therefore n(Yellow\ only)$ ie $n(\overline{blue}) = 20$



b $P(Yellow \cap Blue) = \dfrac{n(Yellow \cap Blue)}{n(Yellow \cup Blue)}$

$$= \frac{10}{38}$$

$$= \frac{5}{19}$$

Page 31 questions

Probability and set diagrams

5 **a** $P(X) = \frac{n(X)}{n(U)} = \frac{4}{30}$

$= \frac{2}{15}$

$= 13\frac{1}{3}\%$

b $P(Y) = \frac{n(Y)}{n(U)} = \frac{13}{30}$

$= 43\frac{1}{3}\%$

c $P(X \cap Y) = \frac{n(X \cap Y)}{n(U)}$

$= \frac{2}{30}$

$= \frac{1}{15}$ or $6\frac{2}{3}\%$

d $P(X \cup Y) = \frac{4}{15} + \frac{15}{30} - \frac{1}{15}$

$= \frac{7}{10}$

$= 70\%$

e $P(Y - X) = \frac{13}{30}$

$= 43\frac{1}{3}\%$

Page 32 questions

Probability and set diagrams

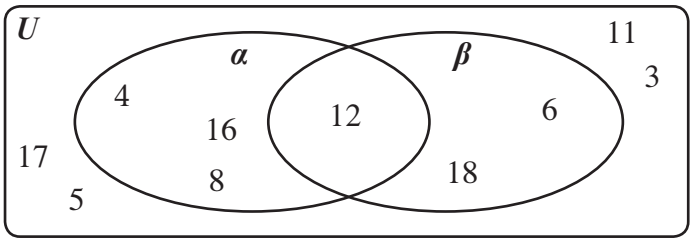
6 **a** $P(\overline{A}) = \frac{n(\overline{A})}{n(A \cup B)} = \frac{8}{17}$

b $P(\overline{C}) = \frac{n(\overline{C})}{n(U)} = \frac{7}{14} = 50\%$

c $P(\overline{Y}) = \frac{n(\overline{Y})}{n(U)} = \frac{12}{21} = \frac{4}{7}$

d $P(\overline{Q}) = \frac{n(\overline{Q})}{n(U)} = \frac{15}{20} = 75\%$

7



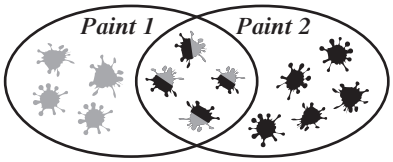
Page 32 questions

Probability and set diagrams

8 **a** $P(\overline{Paint\ 2}) = \dfrac{n(\overline{Paint\ 2})}{n(Paint\ drops)} = \dfrac{4}{13}$ or $P(\overline{Paint\ 2}) = 1 - P(Paint\ 2)$

$= 1 - \dfrac{9}{13} = \dfrac{4}{13}$

b



The drop of paint 1 mixes with a drop of paint 2, meaning that the new mixed drop is recorded where Paint $1 \cap$ Paint 2.
So the number of drops with/without any paint 2 in them does not change.

$\therefore P(\overline{Paint\ 2}) = \dfrac{n(\overline{Paint\ 2})}{n(Paint\ drops)} = \dfrac{4}{13}$,

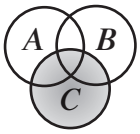
so probability does not change.

Page 34 questions

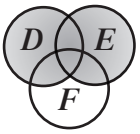
More Venn diagrams

1

a C

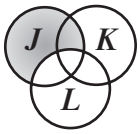


b $D \cup E$

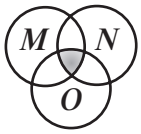


c $G \cap H$

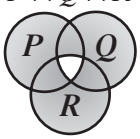
d $J - L$



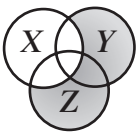
e $M \cap N \cap O$



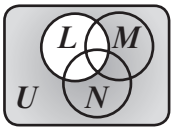
f $\overline{P \cap Q \cap R}$



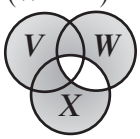
g $\overline{X}$



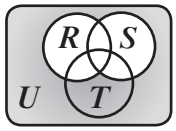
h $\overline{L}$



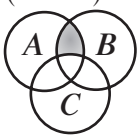
i $\overline{(W \cap X)}$



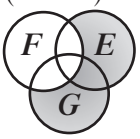
j $\overline{(R \cup S)}$



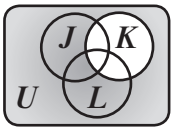
k $(A \cap B) - C$



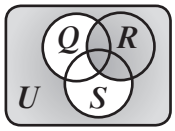
l $(E \cup G) - F$



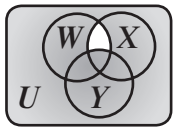
m $\overline{K - J}$



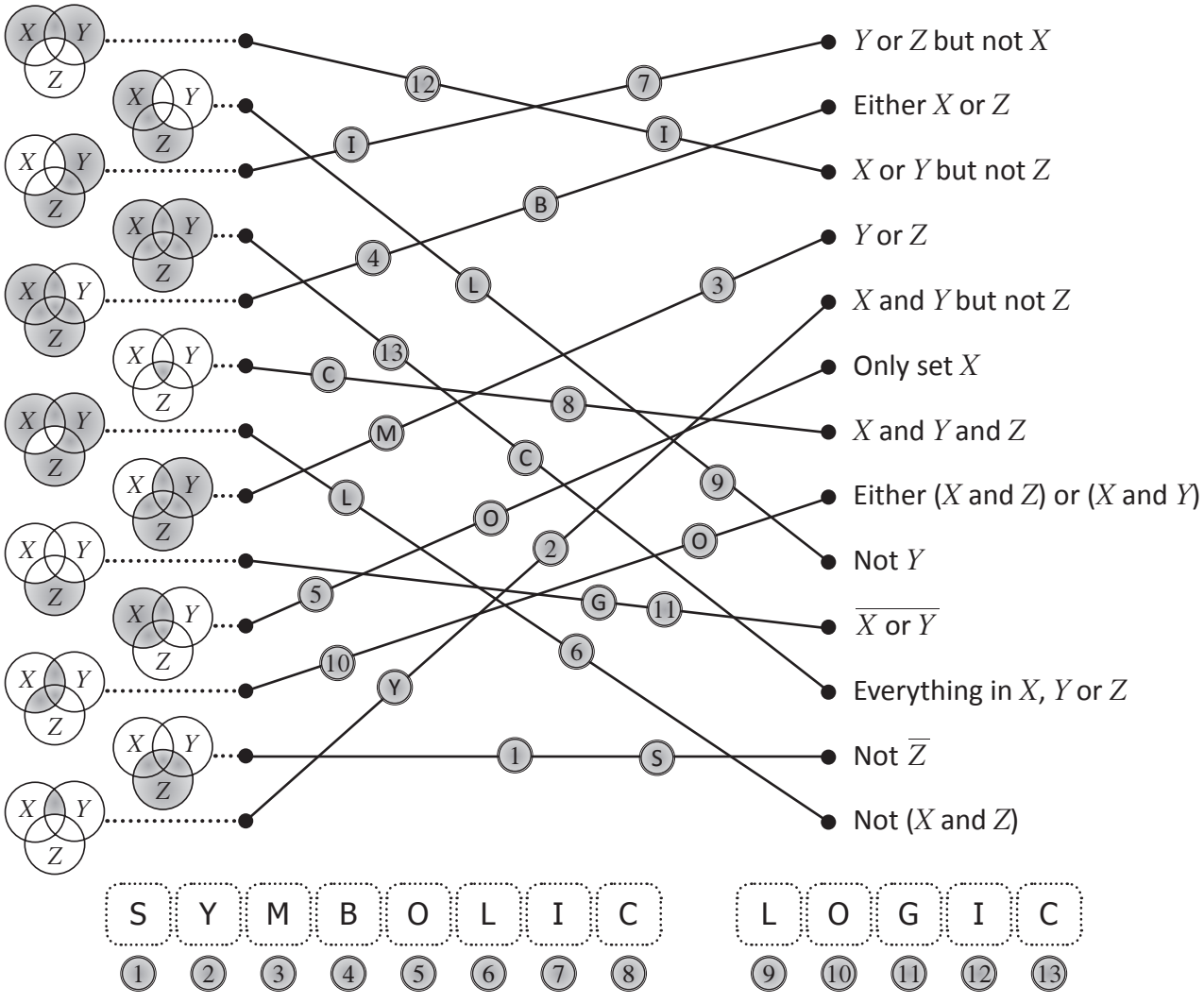
n $\overline{(Q \cup S) - R}$



o $\overline{(W \cap X) - Y}$



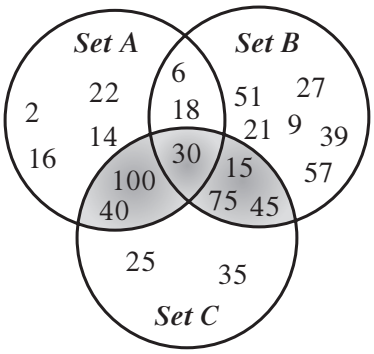
2



Page 35 questions

More Venn diagrams

3



a Set $A = \{2, 6, 14, 16, 18, 22, 30, 40, 100\}$

Set $B = \{6, 9, 15, 18, 21, 27, 30, 39, 45, 51, 57, 75\}$

Set $C = \{15, 25, 30, 35, 40, 45, 75, 100\}$

b $n(Set\ A) =$ 9 $n(Set\ B) =$ 12 $n(Set\ C) =$ 8

c $n(A \cup B \cup C) =$ 20

d (i) Does $n(A \cup B \cup C) = n(Set\ A) + n(Set\ B) + n(Set\ C)$? ☐ Yes ☒ No

(ii) There are members that belong to more than one set. So if the totals of each set were added together, some members would get counted more than once.

e (i) $(A \cup C) = \{2, 6, 14, 15, 16, 18, 22, 25, 30, 35, 40, 45, 75, 100\}$ $\therefore n(A \cup C) =$ 14
(All the different members in set A or set C)

(ii) $(B \cap C) = \{15, 30, 45, 75\}$ $\therefore n(B \cap C) =$ 4
(All the members that are in set B and set C)

(iii) $(A \cap B \cap C) = \{30\}$ $\therefore n(A \cap B \cap C) =$ 1
(All the members that can be found in set A or set B or set C or all)

(iv) $(C - A) = \{15, 25, 30, 35, 45, 75\}$ $\therefore n(C - A) =$ 5
(All the members in set C that are not also members of set A)

(v) $(\overline{A \cup B}) = \{25, 35\}$ $\therefore n(\overline{A \cup B}) =$ 2
(All the members that are not in Set A or Set B)

f (i) $P(A \cup C) = \dfrac{n(A \cup C)}{n(A \cup B \cup C)} = \dfrac{14}{20} = 70\%$

(ii) $P(B \cap C) = \dfrac{n(B \cap C)}{n(A \cup B \cup C)} = \dfrac{4}{20} = 20\%$

(iii) $P(B) = \dfrac{n(B)}{n(A \cup B \cup C)} = \dfrac{12}{20} = 60\%$

(iv) $P(\overline{B}) = \dfrac{n(B \cap C)}{n(A \cup B \cup C)} = 1 - P(B) = \dfrac{8}{20} = 40\%$

(v) $P(\overline{A \cap C}) = \dfrac{n(A \cap C)}{n(A \cup B \cup C)} = \dfrac{17}{20} = 85\%$

(vi) $P(A \cap B \cap C) = \dfrac{n(A \cap B \cap C)}{n(A \cup B \cup C)} = \dfrac{1}{20} = 5\%$

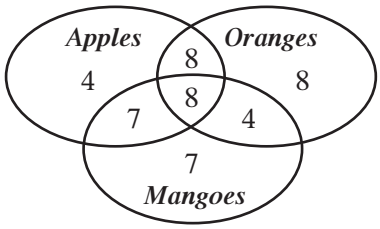
(vii) $P((A \cap C)$ or $(B \cap C)) = \dfrac{n((A \cap C) \text{ or } (B \cap C))}{n(A \cup B \cup C)} = \dfrac{5}{20} = 25\%$

(viii) $P(A$ or C but not $B) = \dfrac{n(A \text{ or } C \text{ but not } B)}{n(A \cup B \cup C)} = \dfrac{8}{20} = 40\%$

Page 36 questions

More Venn diagrams

4 **a**



b (i) $n(Apples) = 27$

(ii) $n(Oranges) = 28$

(iii) $n(Mangoes) = 26$

$\therefore$ The most liked fruit is the Orange

c $$P(Oranges) = \frac{n(Oranges)}{n(Students\ surveyed)} = \frac{28}{4+4+7+7+8+8+8}$$

$$= \frac{28}{46}$$

$$= \frac{14}{23}$$

d $$P(\overline{Apples}) = \frac{n(\overline{Apples})}{n(Students\ surveyed)} = \frac{19}{46}$$

$$= 0.41 \text{ (to 2 d.p.)}$$

e (i) Has the number of students who don't like apples ☐ changed ☒ stayed the same?

(ii) The probability of selecting someone who does not like apples will decrease because the number of students in the survey has increased, so the denominator is larger in the probability fraction.

f $n(Oranges) = 34$

$$P(Oranges) = \frac{n(Oranges)}{n(Students\ surveyed)} = \frac{34}{46+14}$$

$$= \frac{34}{60}$$

$$= \frac{17}{30}$$

$P(\overline{Apples}) = 19$

$$P(\overline{Apples}) = \frac{n(\overline{Apples})}{n(Students\ surveyed)} = \frac{19}{46+14}$$

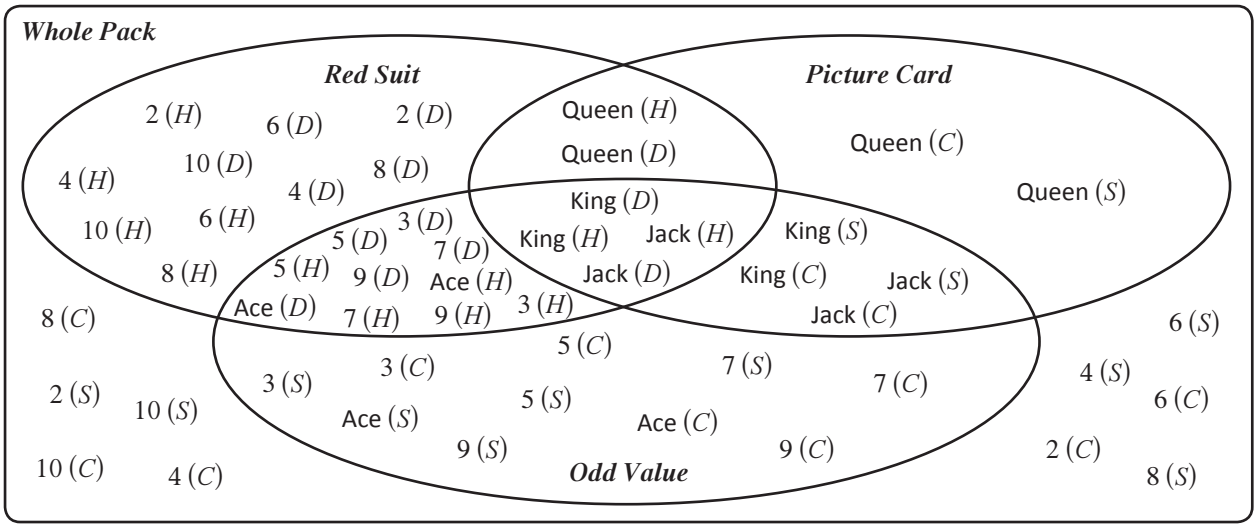
$$= \frac{19}{60}$$

$$= 0.32 \text{ (to 2 d.p.)}$$

Page 37 questions

More Venn diagrams

5 **a**



b (i) $P(Red\ suit)$

$$\frac{n(Red\ suit)}{n(Whole\ pack)} = \frac{26}{52} = 0.50$$

(ii) $P(Picture\ card)$

$$\frac{n(Picture\ cards)}{n(Whole\ pack)} = \frac{12}{52} \approx 0.23$$

(iii) $P(Red\ suit \text{ or } Odd\ valued) = P(Red \cup Odd)$

$$\frac{n(Red \cup Odd)}{n(Whole\ pack)} = \frac{40}{52} \approx 0.77$$

(iv) $P(Red \cap Odd \cap Picture)$

$$\frac{n(Red \cap Odd \cap Picture)}{n(Whole\ pack)} = \frac{4}{52} \approx 0.08$$

(v) $P(\overline{Red \cup Odd \cup Picture})$

$$\frac{n(Red \cup Odd \cup Picture)}{n(Whole\ pack)} = \frac{10}{52} \approx 0.19$$

(vi) $P(\overline{Picture\ card \cap Odd})$

$$\frac{n(Picture\ Card \cap Odd)}{n(Whole\ pack)} = \frac{44}{52} \approx 0.85$$

(vii) $P(Red\ card - Picture\ card)$

$$\frac{n(Red \text{ or } Odd \text{ but not } Picture)}{n(Whole\ pack)} = \frac{30}{52} \approx 0.58$$

(viii) $P(\overline{Black\ picture\ card})$

$$\frac{n(Black\ picture)}{n(Whole\ pack)} = \frac{46}{52} \approx 0.88$$

www.mathletics.com