

Algebra Basics

Solutions



Curriculum Ready



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First edition printed 2009 in Australia.

A catalogue record for this book is available from 3P Learning Ltd.

ISBN 978-1-921861-25-3

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Page 3 questions

Words and symbols

1 a The variable is b

b The variable is m

c The variable is k

d The variable is a

2

$5 \times w = 30$

$x \div x = 1$

$32 - 2 \times d = 16$

$3 + x =$

$12 \times g =$

$200 \div s = 25$

$3 \times x + 6 =$

3

$11 - x$ if $x = 4$

$4 \times m$ if $m = 5$

$27 \div a$ if $a = 3$

$1 + 3 \times z$ if $z = 3$

20

10

9

7

4 a $c = 8$

b $h = 12$

c $k = 18$

d $y = 6$

Page 5 questions

Multiplication

1 a $2 \times 7 \times k = 14 \times k$
 $= 14k$

b $u \times 1 = u$

c $5 \times r \times p = 5pr$

d $n \times m \times m = m^2n$

e $6 \times b \times 3 \times b = 6 \times 3 \times b \times b$
 $= 18 \times b^2$
 $= 18b^2$

f $4 \times j \times l \times 3 \times k = 4 \times 3 \times j \times l \times k$
 $= 12 \times j \times k \times l$
 $= 12jkl$

Page 5 questions

2 a $4pq = 4 \times p \times q$

b $4a^2 = 4 \times a \times a$

c $3m^2n = 3 \times m \times m \times n$

3 Combo Time!

a
$$\begin{aligned} 3x + 2 &= 3 \times 4 + 2 \\ &= 12 + 2 \\ &= 14 \end{aligned}$$

b
$$\begin{aligned} 15 - 2b &= 15 - 2 \times 6 \\ &= 15 - 12 \\ &= 3 \end{aligned}$$

c
$$\begin{aligned} 3 \times 5g &= 3 \times 5 \times 2 \\ &= 30 \end{aligned}$$

d
$$\begin{aligned} 4m^2 &= 4 \times 3^2 \\ &= 4 \times 9 \\ &= 36 \end{aligned}$$

Page 7 questions

Division

1 a $2 \div d = \frac{2}{d}$

b $a \div c = \frac{a}{c}$

c $5 \div (r + 3) = \frac{5}{r + 3}$

d $(y + z) \div z = \frac{y + z}{z}$

2 a $\frac{w}{4} = w \div 4$

b $\frac{c}{3 + a} = c \div (3 + a)$

c $\frac{6}{3x + 2} = 6 \div (3x + 2)$

d $\frac{x - y}{v + w} = (x - y) \div (v + w)$

3 a
$$\begin{aligned} \frac{2a}{6} &= \frac{a}{3} \\ &= a \div 3 \end{aligned}$$

b
$$\begin{aligned} \frac{6b}{12c} &= \frac{b}{2c} \\ &= b \div 2c \end{aligned}$$

c
$$\begin{aligned} \frac{15x}{20y} &= \frac{3x}{4y} \\ &= 3x \div 4y \end{aligned}$$

d
$$\begin{aligned} \frac{4(m + n)}{12p} &= \frac{m + n}{3p} \\ &= (m + n) \div 3p \end{aligned}$$

Page 8 questions

Mixed simplifying concepts

$$\text{1 a } 5 \times a \div 4 = 5a \div 4 \\ = \frac{5a}{4}$$

$$\text{b } 1 \times m \div (4 + n) = m \div (4 + n) \\ = \frac{m}{4 + n}$$

$$\text{c } n \times m \div (b \times a \times c) = mn \div abc \\ = \frac{mn}{abc}$$

$$\text{d } (8 \times 2p) \div (3 \times 3q) = 16p \div 9q \\ = \frac{16p}{9q}$$

$$\text{e } x \times x \div (y + 2x) = x^2 \div (y + 2x) \\ = \frac{x^2}{y + 2x}$$

$$\text{f } d \times f \times d \div (11 + f \times e) = d^2 f \div (11 + ef) \\ = \frac{d^2 f}{11 + ef}$$

$$\text{2 a } \frac{2d}{3} = 2d \div 3 \\ = 2 \times d \div 3$$

$$\text{b } \frac{a+4}{b} = (a+4) \div b$$

$$\text{c } \frac{q-r}{9q} = (q-r) \div 9q \\ = (q-r) \div (9 \times q)$$

$$\text{d } \frac{l^2}{j-k} = l^2 \div (j-k) \\ = l \times l \div (j-k)$$

$$\text{e } \frac{5b^2}{a^2+2b} = 5b^2 \div (a^2+2b) \\ = 5 \times b \times b \div (a \times a + 2 \times b)$$

$$\text{f } \frac{7xyz}{x+7y} = 7xyz \div (x+7y) \\ = 7 \times x \times y \times z \div (x+7 \times y)$$

Page 10 questions

Phrases as algebraic expressions

$$\text{a } n + 7$$

$$\text{b } 9 - n$$

$$\text{c } 6 \times n + 1 = 6n + 1$$

$$\text{d } 4 \times n = 4n$$

Page 10 questions

1 e $(n + 2) \div 3 = \frac{n+2}{3}$

f $n^2 - 6$

g $(n - 5) \times 2 = 2(n - 5)$

h $2 \times n - 8 = 2n - 8$

i $10 + n \div 2 = 10 + \frac{n}{2}$

j $n \times (n + 5) = n(n + 5)$

Page 11 questions

2 a **Correct** Incorrectb **Correct** **Incorrect** expression should be: $n - 4$ c **Correct** Incorrectd **Correct** **Incorrect** expression should be: $(4 + n) \div 9$ or $\frac{4+n}{9}$ e **Correct** **Incorrect** expression should be: $(n \div 5) + n$ or $\frac{n}{5} + n$ f **Correct** Incorrectg **Correct** **Incorrect** expression should be: $n + (3 - \frac{n}{2})$ h **Correct** Incorrecti **Correct** **Incorrect** expression should be: $3 \times n^2 = 3n^2$ j **Correct** Incorrect

Page 13 questions

Addition and subtraction

$$\textcircled{1} \quad \text{a} \quad a + 9a = 1a + 9a \\ = 10a$$

$$\text{b} \quad 3u + 5u = 8u$$

$$\text{c} \quad 14r - 9r = 5r$$

$$\text{d} \quad 4g - 7g = -3g$$

$$\text{e} \quad 6m - 8m = -2m$$

$$\text{f} \quad -11x + 2x = -9x$$

$$\text{g} \quad 7y + 2y + 4y = 13y$$

$$\text{h} \quad 30p - 15p - 10p = 5p$$

$$\textcircled{2} \quad \text{a} \quad \boxed{13m} + 9n \boxed{+12m} = 13m + 12m + 9n \\ \quad \quad \quad \uparrow \text{Like terms} \uparrow \quad = 25m + 9n$$

$$\text{b} \quad 14a \boxed{+b} \boxed{+10b} = 14a + 11b \\ \quad \quad \quad \uparrow \text{Like} \uparrow \\ \quad \quad \quad \text{terms}$$

$$\text{c} \quad 16x \boxed{+9y} \boxed{+15y} = 16x + 24y \\ \quad \quad \quad \uparrow \text{Like} \uparrow \\ \quad \quad \quad \text{terms}$$

$$\text{d} \quad \boxed{9d} - 5c \boxed{-3d} = 9d - 3d - 5c \\ \quad \quad \quad \uparrow \text{Like terms} \uparrow \quad = 6d - 5c$$

$$\text{e} \quad \boxed{7e} \boxed{+11e} + 2a = 18e + 2a \\ \quad \quad \quad \uparrow \text{Like} \uparrow \\ \quad \quad \quad \text{terms}$$

$$\text{f} \quad \boxed{13g} \boxed{-15g} - 4h = -2g - 4h \\ \quad \quad \quad \uparrow \text{Like} \uparrow \\ \quad \quad \quad \text{terms}$$

Page 15 questions

Grouping like terms

$$\begin{array}{c}
 \text{1 a } 9a + 3b + a + 4b = 9a + a + 3b + 4b \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \\ \text{Like terms} \end{array} \\
 = 10a + 7b
 \end{array}$$

$$\begin{array}{c}
 \text{b } 4p^2 + 3p + 19p + 7p^2 = 4p^2 + 7p^2 + 3p + 19p \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \\ \text{Like terms} \end{array} \\
 = 11p^2 + 22p
 \end{array}$$

$$\begin{array}{c}
 \text{c } n - 11m - n - 12m = n - n - 11m - 12m \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \\ \text{Like terms} \end{array} \\
 = -23m
 \end{array}$$

$$\begin{array}{c}
 \text{d } 3y - 5x + y - 8x = 3y + y - 5x - 8x \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \\ \text{Like terms} \end{array} \\
 = 4y - 13x
 \end{array}$$

$$\begin{array}{c}
 \text{e } 9p - 4q + 3p + 12q = 9p + 3p - 4q + 12q \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \\ \text{Like terms} \end{array} \\
 = 12p + 8q
 \end{array}$$

$$\begin{array}{c}
 \text{e } 14a^2 + 4b - 3a + 2a^2 = 14a^2 + 2a^2 + 4b - 3a \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \\ \text{Like terms} \end{array} \\
 = 16a^2 + 4b - 3a
 \end{array}$$

Combo time!

$$\begin{array}{c}
 \text{2 a } 11y \div (2y + 2x - y) = 11y \div (y + 2x) \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \end{array} \\
 = \frac{11y}{y + 2x}
 \end{array}$$

$$\begin{array}{c}
 \text{b } (7p^2 - 5p - 8p^2) \div 12 = (-p^2 - 5p) \div 12 \\
 \begin{array}{c} \text{Like terms} \\ \downarrow \quad \downarrow \\ \uparrow \quad \uparrow \end{array} \\
 = \frac{-p^2 - 5p}{12}
 \end{array}$$

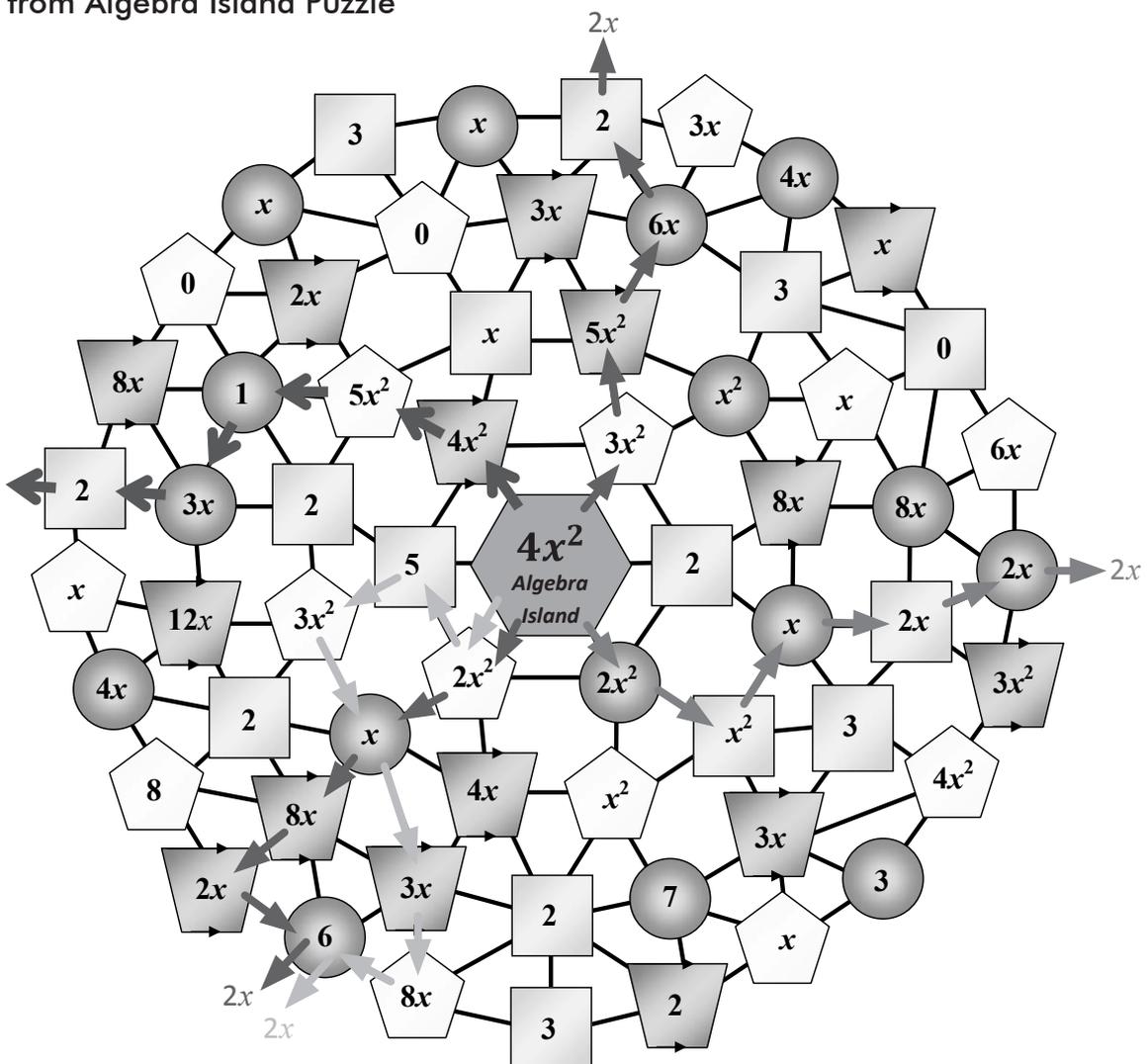
Page 15 questions

3 a $(\overset{\text{Like terms}}{\underbrace{2x - 3y + 2x}}) \div (\overset{\text{Like terms}}{\underbrace{4x + 3x - 2y}}) = (4x - 3y) \div (7x - 2y)$
 $= \frac{4x - 3y}{7x - 2y}$

b $(2 \times 4a + 3 \times 2b) \div (3 \times a \times a + 2a^2) = (8a + 6b) \div (\overset{\text{Like terms}}{\underbrace{3a^2 + 2a^2}})$
 $= (8a + 6b) \div 5a^2$
 $= \frac{8a + 6b}{5a^2}$

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Escape from Algebra Island Puzzle



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Bringing all the previous concepts together

$$\begin{aligned} \text{1 a } 4v + 2 &= 4 \times 4 + 2 \\ &= 18 \end{aligned}$$

$$\begin{aligned} \text{b } 24 \div 2v &= 24 \div (2 \times 4) \\ &= 24 \div 8 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{c } 10 - \frac{v}{4} &= 10 - \frac{4}{4} \\ &= 10 - 1 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{d } \frac{2v+6}{7} &= (2v+6) \div 7 \\ &= (2 \times 4 + 6) \div 7 \\ &= 14 \div 7 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{2 a } a + 2b &= -2 + 2 \times 5 \\ &= -2 + 10 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{b } 3b - 6a &= 3 \times 5 - 6 \times (-2) \\ &= 15 - (-12) \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{c } \frac{24}{a+b} &= 24 \div (a+b) \\ &= 24 \div (-2+5) \\ &= 24 \div 3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{d } \frac{a^2b}{4} &= a^2b \div 4 \\ &= ((-2)^2 \times 5) \div 4 \\ &= (4 \times 5) \div 4 \\ &= 20 \div 4 \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{3 a } \begin{array}{cccc} & \swarrow \text{Like terms} \swarrow & & \\ \textcircled{c} & \textcircled{+d} & \textcircled{+2c} & \textcircled{+3d} \\ \uparrow \text{Like terms} \uparrow & & & \end{array} &= 3c + 4d \\ &= 3 \times 6 + 4 \times 9 \\ &= 18 + 36 \\ &= 54 \end{aligned}$$

$$\begin{aligned} \text{b } \begin{array}{cccc} & \swarrow \text{Like terms} \swarrow & & \\ \textcircled{2c} & \textcircled{+d} & \textcircled{+3c} & \textcircled{-d} \\ \uparrow \text{Like terms} \uparrow & & & \end{array} &= 5c \\ &= 5 \times 6 \\ &= 30 \end{aligned}$$

$$\begin{aligned} \text{c } \frac{2d-c}{d-c} &= (2d-c) \div (d-c) \\ &= (2 \times 9 - 6) \div (9 - 6) \\ &= 12 \div 3 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{d } (c-d) \times (2c-d) &= (c-d) \times (2c-d) \\ &= (6-9) \times (2 \times 6 - 9) \\ &= (-3) \times 3 \\ &= -9 \end{aligned}$$

Page 20 questions

$$\begin{aligned}
 \text{4 a } 2x + y + z &= 2 \times 6 + 3 + (-8) \\
 &= 12 + 3 - 8 \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 \text{b } 3z - xy &= 3 \times (-8) + 6 \times 3 \\
 &= -24 + 18 \\
 &= -6
 \end{aligned}$$

$$\begin{aligned}
 \text{c } x^2 - yz &= 6^2 - (3 \times (-8)) \\
 &= 36 - (-24) \\
 &= 60
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \frac{4y}{x+z} &= 4y \div (x+z) \\
 &= 4 \times 3 \div (6 + (-8)) \\
 &= 12 \div (-2) \\
 &= -6
 \end{aligned}$$

$$\text{5 } \frac{a(a+2b)^2}{(b-a)^2} = a(a+2b)^2 \div (b-a)^2 \quad \text{when } a = 2, b = -4$$

$$\begin{aligned}
 &= 2 \times (2 + 2 \times (-4))^2 \div (-4 - 2)^2 \\
 &= 2 \times (-6)^2 \div (-6)^2 \\
 &= 2 \times 36 \div 36 \\
 &= 2
 \end{aligned}$$

$$\text{6 } \left(\frac{(x-y)^2}{(y-x)^2} \right)^2 = ((x-y)^2 \div (y-x)^2)^2 \quad \text{when } x = -1, y = -5$$

$$\begin{aligned}
 &= ((-1 - (-5))^2 \div (-5 - (-1))^2)^2 \\
 &= ((-1 + 5)^2 \div (-5 + 1)^2)^2 \\
 &= (4^2 \div (-4)^2)^2 \\
 &= (16 \div 16)^2 \\
 &= (1)^2 \\
 &= 1
 \end{aligned}$$

Page 22 questions

Tables of Values

1 a $u = v + 2$

v	0	1	2	3	4
u	2	3	4	5	6

b $c = 2d$

d	0	1	2	3	4
c	0	2	4	6	8

c $g = 4h - 3$

h	1	2	3	4	5
g	1	5	9	13	17

d $y = \frac{x}{2} + 1$

x	2	4	6	8	10
y	2	3	4	5	6

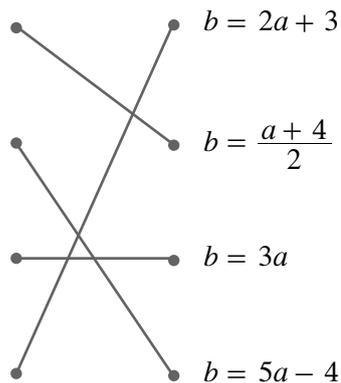
2

a	0	2	4	6	8
b	2	3	4	5	6

a	1	2	3	4	5
b	1	6	11	16	21

a	0	1	2	3	4
b	0	3	6	9	12

a	0	1	2	3	4
b	3	5	7	9	11



3 a Rule: $y = x + 5$

x	0	1	2	3	4
y	5	6	7	8	9

b Rule: $n = 4 \times m$ or $n = 4m$

m	0	1	2	3	4
n	0	4	8	12	16

c Rule: $q = p - 3$

p	0	1	2	3	4
q	-3	-2	-1	0	1

d Rule: $d = 4c - 5$

c	0	1	2	5	6
d	-5	-1	3	15	19

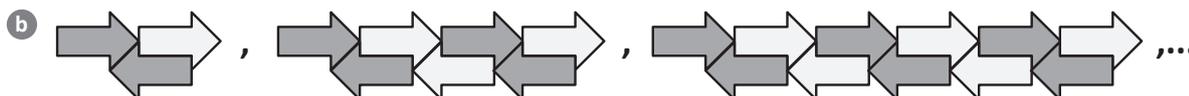
Page 24 questions

Number patterns



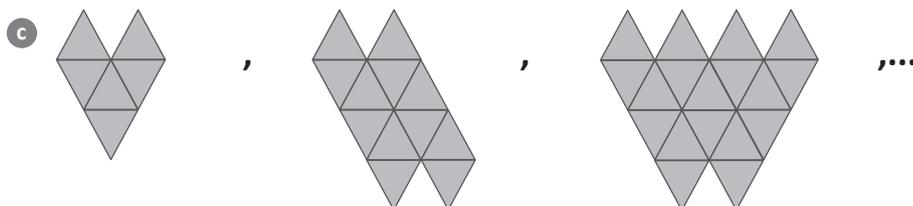
(i) Starting with one smiley face in the first diagram, 2 smiley faces are added to each diagram every time

(ii) Number pattern for first five smiley diagrams: 1, 3, 5, 7, 9, ...



(i) Starting with three arrows in the first diagram, 4 arrows are added to each diagram every time

(ii) Number pattern for first five arrow diagrams: 3, 7, 11, 15, 19, ...



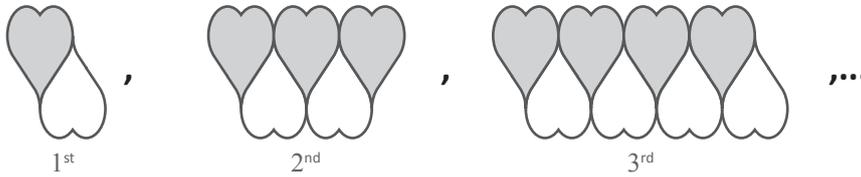
(i) Starting with 6 triangles to form the first diagram, 6 triangles are added to each diagram every time.

(ii) Number pattern for first five arrow diagrams: 6, 12, 18, 24, 30, ...

Page 25 questions

Number patterns

2 a

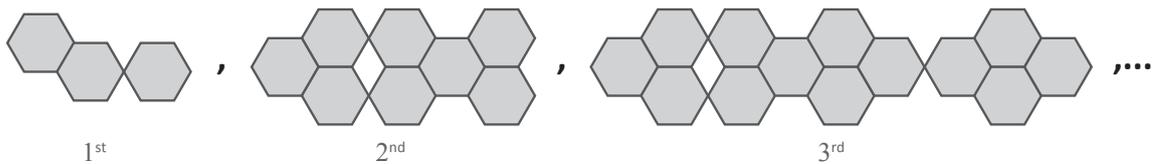


(i)

Diagram number	1	2	3	4
Number of hearts	2	5	8	11

(ii) Number of hearts needed for the 7th diagram = 20

b

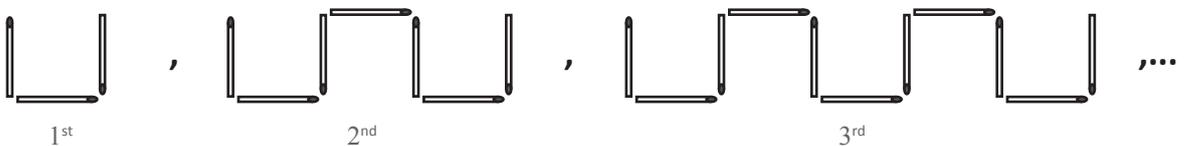


(i)

Diagram number	1	2	3	4
Number of hexagons	3	8	13	18

(ii) Number of hexagons needed for the 7th diagram = 33

c



(i)

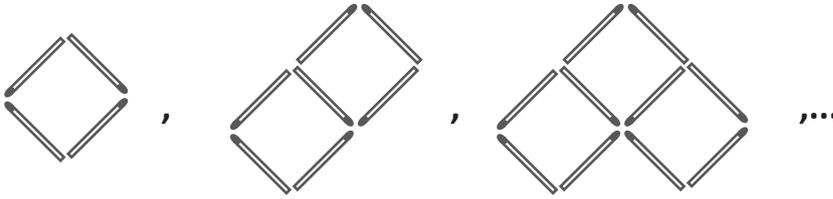
Diagram number	1	2	3	4
Number of matchsticks	3	7	11	15

(ii) Number of matchsticks needed for the 7th diagram = 27

Page 27 questions

Modelling Number Patterns

1 a

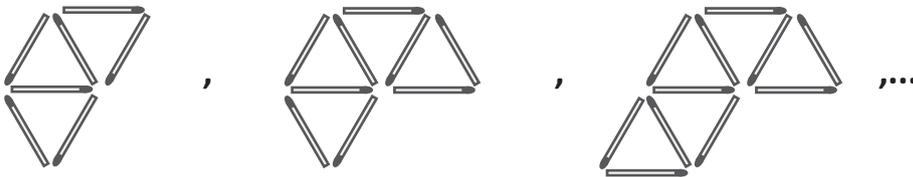


Number of squares (s)	1	2	3
Number of matchsticks (m)	4	7	10

General rule:

$$m = 3 \times s + 1$$

b



Number of triangles (t)	3	4	5
Number of matchsticks (m)	7	9	11

General rule:

$$m = 2t + 1$$

c

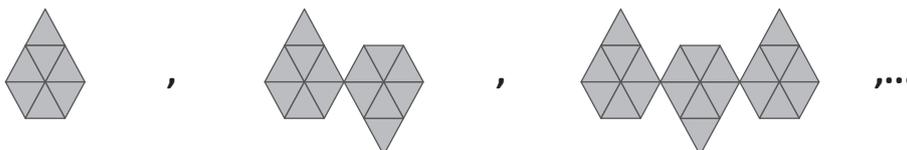


Number of grey rings (r)	1	2	3
Number of circle drawn (c)	2	3	4

General rule:

$$c = 1r + 1$$

d



Number of pentagonal shapes (p)	1	2	3
Number of triangles (t)	7	14	21

General rule:

$$t = 7p$$

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More number pattern modelling

1 a

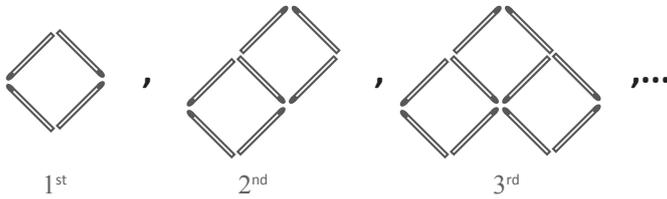


Diagram number (n)	1	2	3
Number of matchsticks (m)	4	7	10

General rule:

$$m = 3 \times n + 1$$

b

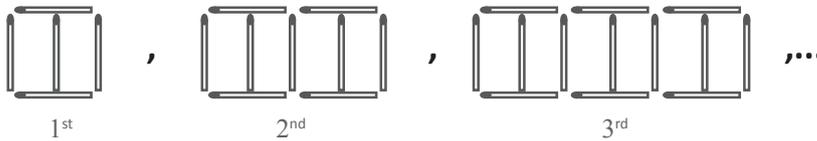


Diagram number (n)	1	2	3
Number of matchsticks (m)	5	9	13

General rule:

$$m = 4n + 1$$

c

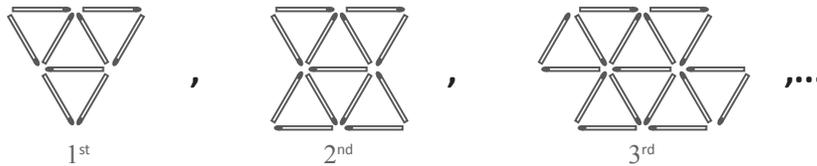


Diagram number (n)	1	2	3
Number of matchsticks (m)	9	13	17

General rule:

$$m = 4n + 5$$

d

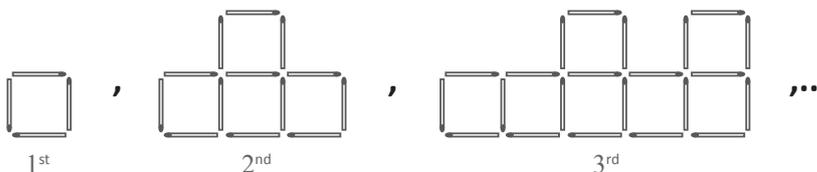


Diagram number (n)	1	2	3
Number of matchsticks (m)	4	13	22

General rule:

$$m = 9n - 5$$

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More number pattern modelling

2 a



Diagram number (n)	1	2	3
Number of snow flakes (s)	2	4	6

General rule:

$$s = 2 \times n$$

b

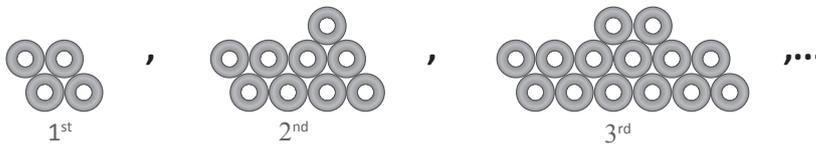


Diagram number (n)	1	2	3
Number of tyres (t)	4	9	14

General rule:

$$t = 5n - 1$$

c



Diagram number (n)	1	2	3
Number of dots (d)	5	9	13

General rule:

$$d = 4 \times n + 1$$

d

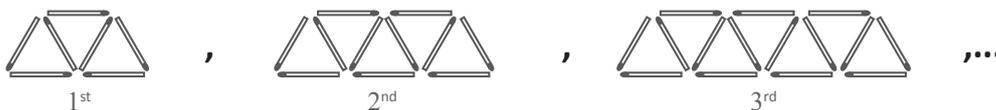


Diagram number (n)	1	2	3
Number of triangles (t)	3	5	7

General rule:

$$t = 2 \times n + 1$$

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Using the general rule

1 a $s = 2g$

Niamh scored: $s = 2 \times 8$
 $= 16$ points

b $c = 5m - 3$

The number of chickens that had crossed the road: $c = 5 \times 7 - 3$
 $= 32$ chickens

c $s = 2c + 1$

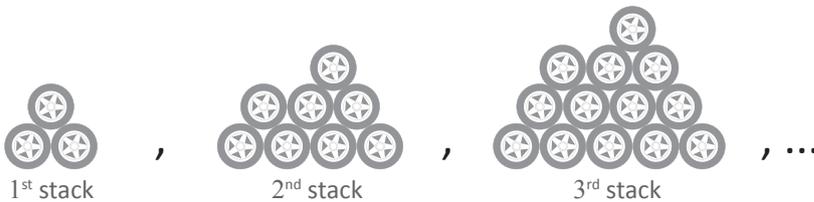
The number of shirts tried on: $s = 2 \times 12 + 1$
 $= 25$ shirts

d $v = \frac{d}{3}$

The number of vegetarian meals ordered: $v = \frac{36}{3}$
 $= 12$ vegetarian meals

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2

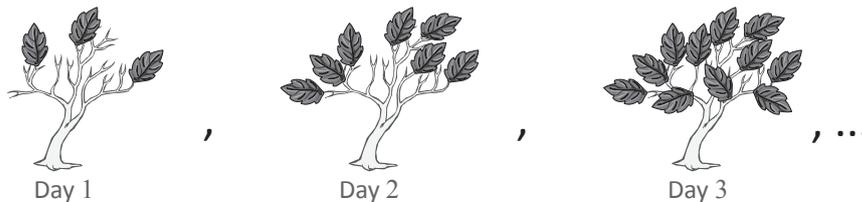


n	1	2	3
t	3	8	13

General rule:
 $t = 5n - 2$

Tyres in the 12th stack:
 $t = 5 \times 12 - 2$
 $= 58$

3



n	1	2	3
l	3	7	11

General rule:
 $l = 4n - 1$

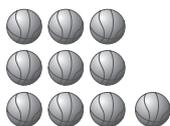
Leaves on the 10th day:
 $l = 4 \times 10 - 1$
 $= 39$

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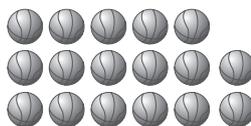
4



Session 1



Session 2



Session 3

n	1	2	3
s	3	10	17

General rule:

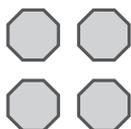
$$s = 7n - 4$$

Good shots in the 8th session:

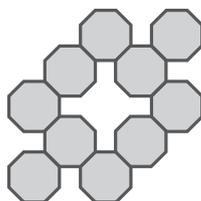
$$\begin{aligned} s &= 7 \times 8 - 4 \\ &= 52 \end{aligned}$$

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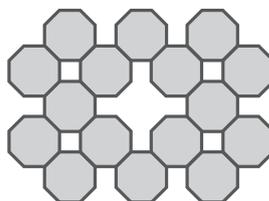
5



1st diagram



2nd diagram



3rd diagram

n	1	2	3
t	4	10	16

General rule:

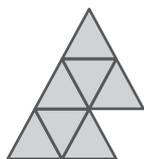
$$t = 6n - 2$$

Tiles laid in the 12th diagram:

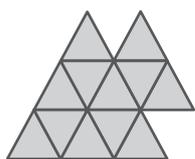
$$\begin{aligned} t &= 6 \times 12 - 2 \\ &= 70 \end{aligned}$$

Remember me?

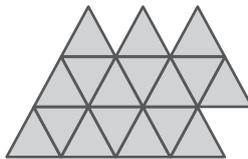
6



7 triangles



12 triangles



17 triangles

n	1	2	3
t	7	12	17

General rule:

$$t = 5n + 2$$

Triangles in the 15th shape:

$$\begin{aligned} t &= 5 \times 15 + 2 \\ &= 77 \end{aligned}$$



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